Born's Rule

Let us examine Born's rule in more detail. Consider again Young's double-slit experiment shown in Figure 1.4. In Figure 1.7 two detectors are placed after the slit in separate locations. A detector that has detected a photon will be denoted by a white detector, otherwise it will be assumed to be in its initial pre-detection state denoted by a black detector; in Figure 1.7 both detectors are in the pre-detection state and are

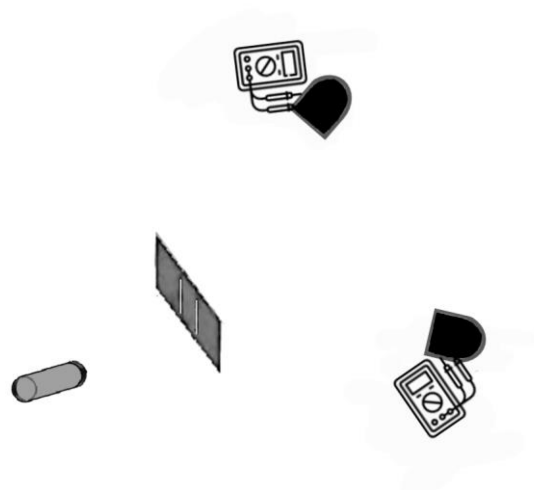

Figure 1.7: Young's double slit with detectors in their ready state as shown in black.

black.

Now, suppose a single photon is incident on the double slit. According to Schrödinger's equation, a wave propagates through the slit and impinges on both detectors as shown in Figure 1.8. As of yet no detection has been made. At this point the wave is physically interacting with *both* detectors. Let us assume that one of the two devices detects the photon, which is denoted by a white detector.

One might initially expect that the wave is still present—consider for example Figure 1.9 (the unitary prediction involves entanglement and is discussed in Chapter 2). However, this is not the case in the Copenhagen interpretation of Bohr and Heisenberg. When a measurement is known to have been made, one invokes the measurement postulate due to Born and formalized further by von Neumann. The situation is shown in Figure 1.10.

The unitary Schrödinger wave ceases to exist after the measurement postulate is applied and the situation of Figure 1.8 is replaced with Figure 1.10, which happens when a measurement occurs in the lower detector. The replacement of Figure 1.8 with Figure 1.10 is a discontinuous change and is often referred to as a quantum jump. There was good reason historically to consider Figure 1.10 versus Figure 1.8 in order to explain measurement. At the time of Schrödinger's discovery of his equation in 1926, it had already been known for over twenty years, going back to Einstein's

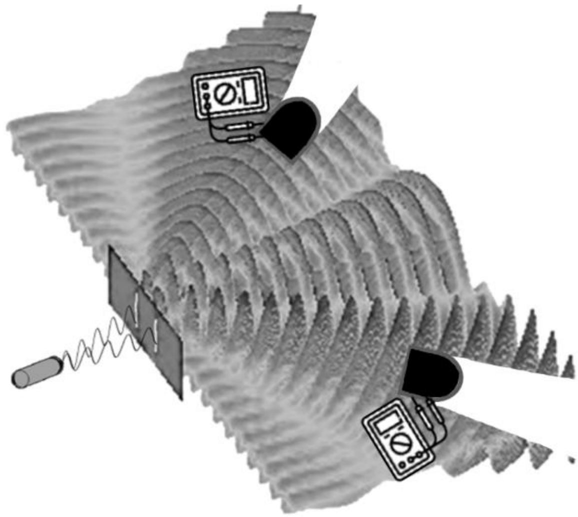

Figure 1.8: Young’s double slit with photon wave propagating via Schrödinger's equation and wave function impinging on both detectors.

discovery, that light exists in quantized energy units called photons. One can see that if Figure 1.8 was considered to be correct, it would contradict the existence of such indivisible units. The detector would have to be said to have absorbed the energy in the photon (by the photoelectric effect proposed by Einstein in 1905), and yet if the wave still existed it would seemingly be able to be absorbed by another detector,

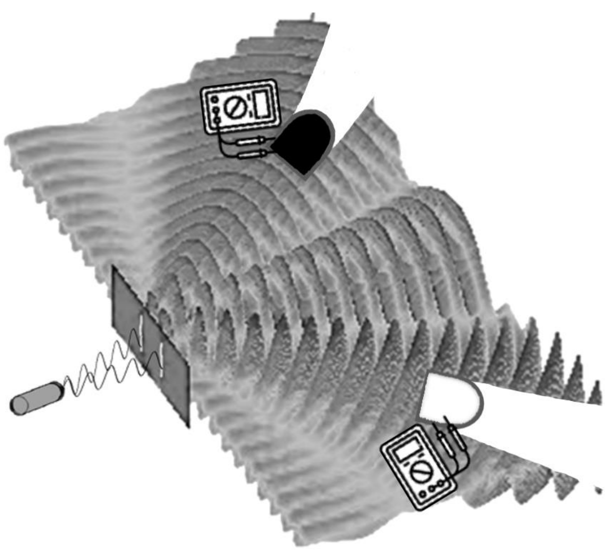

Figure 1.9: Incorrect explanation of Young’s slit with a photon detection shown by white detector; wave and detection exist simultaneously.

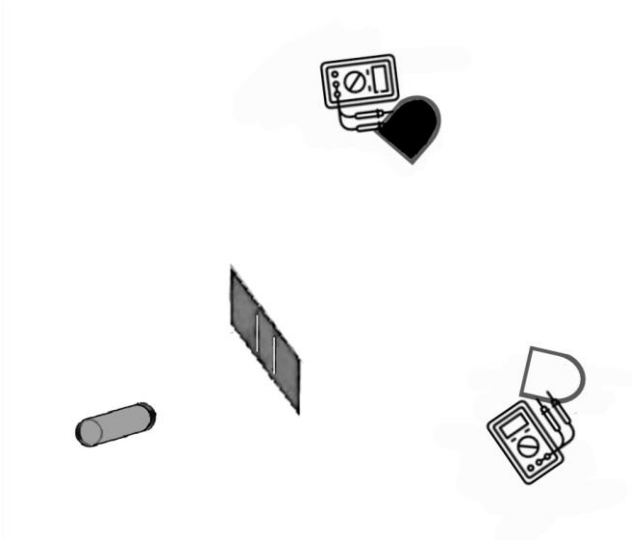

Figure 1.10: State evolution under the measurement postulate where lower detector shown in white, registers a photon.

which would contradict energy conservation.

So far, we have examined the case of a single detection as illustrated by the lower white detector in Figure 1.10. However, it is also possible that the upper detector detects the photon as opposed to the lower detector as shown in Figure 1.11. One sees that there are three possible outcomes of the experiment; it is possible that no detection is made, for which the result would be Figure 1.7, and as well there are two possible outcomes when detections occur—those represented by Figure 1.10 and

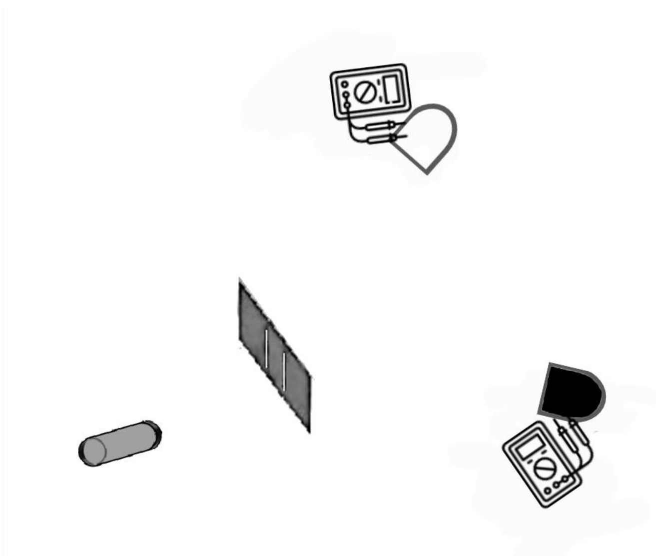

Figure 1.11: Second possible case of detection shown by upper white detector.

Figure 1.11. In 1926, the question became how one mathematically describes the physics of the situation.

In 1926 shortly after learning Schrödinger's equation, Born proposed the statistical nondeterministic interpretation. Born did not ask how Schrödinger's equation could be used to derive the particular wave function that occurs after a collision for individual events, but rather only how probable a specific outcome of the collision would be. Born considered an electron scattering with an atom. The final electron energy would be increased by one quantum at the cost of lowering the energy of the atom. However, the direction of the electron after scattering was uncertain in Schrödinger's description and written in terms of a superposition of outcomes of the atomic unperturbed eigenstates Ψ_m that are weighted by a function $\Phi_{\mathrm{m}}(\alpha,\beta,\gamma)$ where α,β,γ specifies the scattering angles. Born proposed that the probability of any specific outcome would be found when considering the scattered wave function of the form $\sum_m \int \Phi_{\mathrm{m}}(\alpha,\beta,\gamma)\Psi_m$. The probability of finding the electron scattered with particular angles α,β,γ was proposed by Born to be proportional to $|\Phi_{\mathrm{m}}(\alpha,\beta,\gamma)|^2$.

In the case of Young's double-slit, also called an *interferometer*, one can develop a nondeterministic interpretation based on Born's rule as follows. Let the wave function that impinges on the upper detector at x_1 in Figure 1.3 be given as the superposition $\psi\text{`}_L(x_1,t_0) + \psi\text{`}_R(x_1,t_0)$ and on the lower detector as $\psi\text{`}_L(x_2,t_0) + \psi\text{`}_R(x_2,t_0)$. The initial situation shown in Figure 1.8 then jumps to either Figure 1.10 or Figure 1.11 depending on the outcome of the measurement which is determined statistically. The

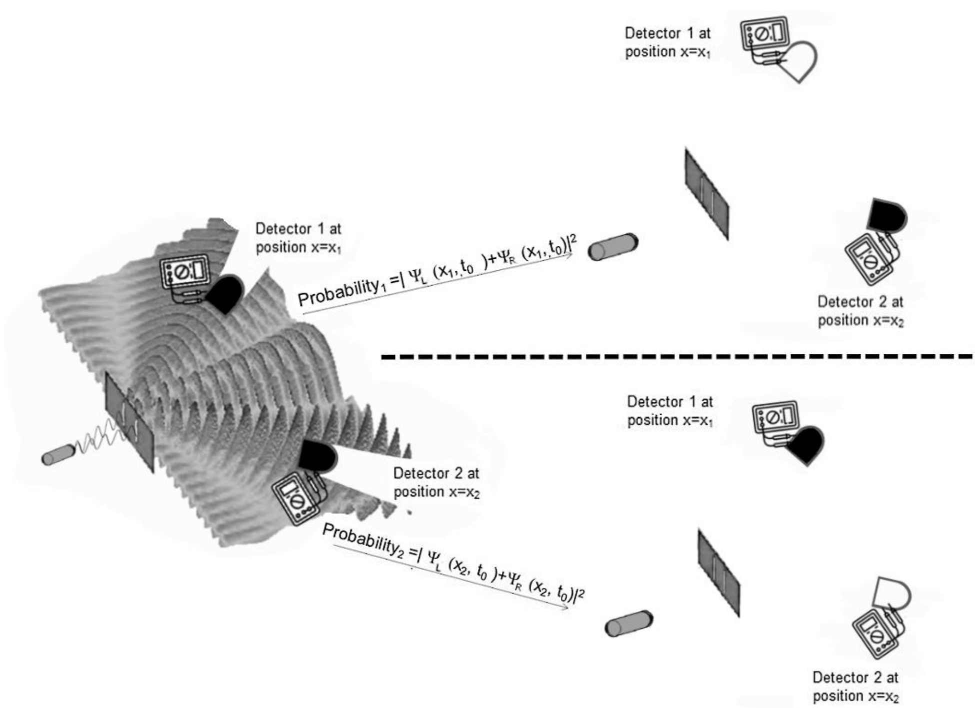

Figure 1.12: Statistical bifurcation of Born's rule.

resulting probability of Figure 1.10 is given by the square of the wave function at $(x_{2,}$ $t_0)$, which is $|\psi`_L(x_2, t_0) + \psi`_R(x_2, t_0)|^2$. Similarly, the resulting probability of the photon being detected by the upper detector in Figure 1.11 is given by $|\psi`_L(x_1, t_0) + \psi`_R(x_1, t_0)|^2$. Upon expanding these expressions, the cross-terms produce interference between L and R. This bifurcation is shown in Figure 1.12.

A question that immediately arises is whether this is simply an approximation due to the lack of knowledge of the phases of the particles accumulated during the collision, or whether the underlying physics is truly nondeterministic. This issue was brought up in Born's original paper [6] and considered by Bohr, Heisenberg, and others. Von Neumann in 1927 published a series of papers that attempted to formalize the theory. Von Neumann introduced a projection postulate which represented measurement by an operator that changes the state when a measurement occurs. The projection postulate represents a discontinuous change of the state, depending on the result of the measurement. Dirac adopted the projection postulate in his 1930 textbook which became the standard presentation of quantum mechanics.