Entropy of Quantum States

The entropy of a state ρ in quantum mechanics is given by the von Neumann entropy

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho) = -\sum_i \lambda_i \log \lambda_i. \quad (2.32)$$

where the λ_i are the eigenvalues of ρ. A pure state $\rho = |\psi\rangle\langle\psi|$ has the minimal value of the entropy, $S(\rho) = 0$. This signifies that further information cannot be obtained for a pure state by repeated measurement over many copies. The entropy of a system obeying the Schrödinger equation always remains constant since the entropy only depends on the eigenvalues of ρ, which cannot change under a unitary transformation.

Thus, given a unitary transformation U and a density matrix ρ, we find

$$S(U^\dagger \rho U) = S(\rho). \quad (2.33)$$

The entropy is thus independent of time under unitary evolution and the information about a state does not change, i.e.,

$$\frac{dS}{dt} = 0. \quad (2.34)$$

The entropies of the subsystems are also of interest since they give information about quantum entanglement for pure states. Details about subsystem entropies in terms of the *Araki-Lieb Inequality* are presented next.