Information Flow in Entangled Composite Systems

The unitary properties of reversibility and constant entropy during evolution impose constraints on what tasks can be accomplished by unitaries and in what way they are able to direct information within entangled composite systems. This provides insight into the capabilities and limitations of unitaries. Consider an example involving three systems *A*, *B*, and *C*, which may be coupled by a variety of unitary interactions. Although the system is completely governed by reversible unitary evolution, let us examine whether configurations are possible which allow only one-way information transfer between subsystems. Information transfer means, for example, that there

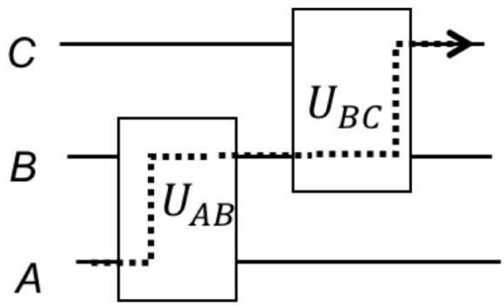

Figure 2.12: Configuration of the only possible unitary evolution of system *ABC* not allowing information flow from *C* to *A* [After Schumacher et al. [53]].

would be no information flow from *C* to *A* if the final state of *A* depended on the initial state of *AB* alone. The only possibility for this occurring within a unitary transformation is the configuration in Figure 2.12 as proved by Schumacher and Westmoreland [53]. This has local interactions of the form $U_{ABC} = (I_A \otimes U_{BC})(U_{AB} \otimes I_C)$ which allows information to flow from *A* to *B* to *C* but prohibits any information from *C* to *A*. It is the local sequencing of interactions—first *AB* and then *BC*—that allows local restrictions in flow but still maintains the complete reversibility of the entire unitary operator U_{ABC} as it must (Exercise 2.11 in the book or kindle version of theQMP).

A unitary transformation must resort to local sequencing of interactions to shuttle and restrict information flow because linearity implies that certain unitary transformations are impossible. A prominent example is the *no-cloning theorem* [54]: no unitary transformation can copy an unknown quantum state $|\psi\rangle$. If this was possible, we could construct the following transformation for unknown states $|\psi_1\rangle$ and $|\psi_2\rangle$ [14, p. 532]

$$U(|\psi_1\rangle \otimes |c\rangle) = |\psi_1\rangle \otimes |\psi_1\rangle \quad (2.43)$$

$$U(|\psi_2\rangle \otimes |c\rangle) = |\psi_2\rangle \otimes |\psi_2\rangle. \quad (2.44)$$

where $|c\rangle$ is the initial state of the cloning machine. Taking the inner product between Equations (2.43) and (2.44) yields $\langle\psi_2|\psi_1\rangle = \langle\psi_2|\psi_1\rangle^2$, which only has the solutions

$\langle\psi_2|\psi_1\rangle = 0$ or 1. Therefore unitary cloning can only be done for orthogonal states and not for a general quantum state. The limits to cloning have also been quantified and there are by now also a variety of other *no-go theorems,* which restrict what can be accomplished by unitaries, including the quantum no-deletion theorem [55], no-broadcasting [56], and no-unconditionally secure quantum bit commitment [57] [58].

The root of the problem for the no-cloning and related theorems is the necessary restriction to having only local sequencing of information flow in unitaries similar to those illustrated in Figure 2.12. Such restrictions also occur in the functioning of quantum gates such as the CNOT gate which carries out photonic entanglement in the polarizing beam splitters [53]. The CNOT cannot simply have a global exchange of information between the control and target qubits as it would then violate no-cloning. Instead, the information flow is locally directional from control to target and this limits what a CNOT or other unitary gates can achieve. However, it should be emphasized that local information flow from *A* to *C* need not imply a causal relation between *A* and *C*. Because of the linearity of unitary transformations, it is possible to form a superposition of two gates, one of which allows flow from *A* to *C* and the other from *C* to *A*. Such a composite gate of quantum-causal superposition would have the remarkable property of having no causal order between *A* and *C*, since signals would be sent in each order simultaneously [59]. In an experiment using polarized photons in such a superposition of causal orders, it has even been demonstrated that the photon can be measured without revealing the causal order [60]. This was done by embedding the result of the measurement into the photon itself but, then delaying the extraction of any information about the measurement result until after the photon goes through the entire circuit. This allows a coherent measurement at different times which erases the causal ordering information. The properties of unitary transformations thus allow the possibility of having physical processes without causal ordering, and this is also consistent with the inherent no-signaling property of quantum mechanics in Equation (2.31)