Reversibility

An operator that is unitary is also completely reversible. That is, suppose that at some initial time t_1 the state of the system is $|\psi(t_1)\rangle$ and a unitary operator U is found that evolves the state to $|\psi(t_2)\rangle = U|\psi(t_1)\rangle$. Then there is another unitary operation that can evolve $|\psi(t_2)\rangle$ to $|\psi(t_1)\rangle$ and completely undo the effect of the initial U. In fact, mathematically, this second unitary operation is found by the Hermitian conjugate $U^\dagger$ (found by taking the transpose and complex conjugate $U^\dagger \equiv (U^*)^T$and which is equal to its inverse $U^\dagger = U^{-1}$). That is, $|\psi(t_1)\rangle = U^\dagger|\psi(t_2)\rangle$.

It will be shown later in Equation (2.11) of the theoretical development, that unitary evolution can be decomposed in the simple form of

$$|\psi(t)\rangle = \sum_k a_k(t)|\phi_k\rangle$$

where $\sum_k |a_k(t)|^2 = 1$ and for which $|\phi_k\rangle$ are fixed or stationary states called eigenstates of the Hamiltonian. The coefficients can be represented by

$$a_k(t) = a_k(0)\exp(-iE_k t/\hbar).$$

This shows that during unitary evolution, each eigenstate $|\phi_k\rangle$ undergoes a steady phase rotation that is linear in time, $\theta_k(t) = E_k t/\hbar$, with a rate proportional to the energy E_k. Therefore, unitary time evolution reduces to nothing more than simple linear progression in time, comparable to the inexorable ticking gears of a clock, Figure 2.4. Unitary evolution is akin to a simple reversible mechanical process. This characteristic steady phase rotation also results in the additional unitary properties of

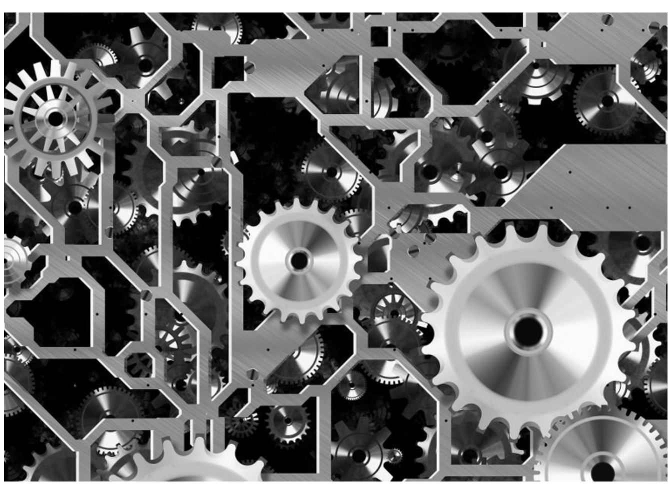

Figure 2.4: Unitary evolution reduces to the simple linear progression in time of the energy eigenstate phases comparable to the inexorable ticking gears of a clock.

reversibility and preservation of information (i.e., entropy) during evolution, which will be further elaborated on in the theoretical development. Could it be possible that the click of the detector in Figure 2.3 can be described by a unitary transformation, despite its apparent lack of resemblance to the inexorable ticking gears of a clock? In Chapter 3, we will discuss the distinguishability between unitaries and measurement.