Two Polarization-Entangled Photons

We have discussed using polarizing beam splitters to create single qubit states involving the horizontal H and vertical V polarization degrees of freedom. Polarizing beam splitters can also be used to create two-qubit entangled states similar to Equation (2.17) as were found to be maximally entangled in the previous section. These are created by inputting two photons into the beam splitter as in Figure 2.10. The two photons can be incident into the same port or into different ports. With two photons now present, the boson statistics of the photons also have to be take into account. The

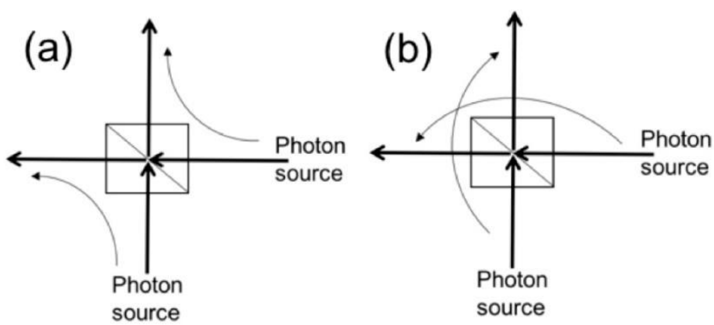


Figure 2.10: Two photons sent into a 50-50 polarizing beam splitter for the cases (a) both are reflected, (b) both are transmitted.

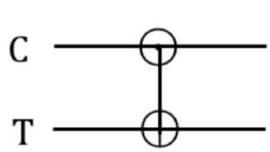
photon has two polarization modes H and V as well as two spatial degrees of freedom a and b, so that the vector describing the two-photon state is given by

$$\Phi = \begin{pmatrix} a_{\mathrm{H}} \\ b_{\mathrm{H}} \\ \mathrm{a}_{\mathrm{V}} \\ \mathrm{b}_{\mathrm{V}} \end{pmatrix} \tag{2.39}$$

where a_i and b_i denote the spatial degrees of freedom associated with polarization i. Since photons are bosons, the total state for the combined polarization and spatial degrees of freedom must be symmetric under permutation of the particles. Using the basis in Equation (2.39), the polarization response from a polarizing beam splitter is identical to that of the controlled-not or CNOT gate used in quantum computing [51] (Exercise 2.8 in the book or kindle version of theQMP):

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2.40}$$

The CNOT gate is a convenient unitary transformation that has the ability to produce two entangled qubits. It's customary to consider one of the qubit inputs to the CNOT as the *control qubit* C and the other as the *target qubit* T. The CNOT unitary transformation functions by flipping the target qubit depending on the state of the control qubit. Thus, it is a unitary operator that changes one qubit conditional on the state of the other. Figure 2.11 shows the standard circuit diagram representing the CNOT and its operation for the possible two-qubit inputs.



$$\begin{pmatrix} |\mathrm{CT}\rangle \longrightarrow |\mathrm{CT}\rangle \\ |\mathrm{HH}\rangle \longrightarrow |\mathrm{HH}\rangle \\ |\mathrm{HV}\rangle \longrightarrow |\mathrm{HV}\rangle \\ |\mathrm{VH}\rangle \longrightarrow |\mathrm{VV}\rangle \\ |\mathrm{VV}\rangle \longrightarrow |\mathrm{VH}\rangle \end{pmatrix}$$

Figure 2.11: Quantum Controlled-Not or CNOT two-qubit unitary gate which flips the target state (T) only if the control state (C) is 1.

As an example, consider the non-entangled polarization state $|\mathrm{CT}\rangle_{in} = (|\mathrm{H}\rangle + |\mathrm{V}\rangle)\otimes|\mathrm{H}\rangle/\sqrt{2}$ as input to the CNOT. From the table in Figure 2.11 the output is found to be the maximally entangled state (Exercise 2.9 in the book or kindle version of theQMP):

$$|\mathrm{CT}\rangle_{\mathrm{out}} = \mathrm{CNOT}\left(\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\otimes|H\rangle\right) = \frac{1}{\sqrt{2}}(|H\rangle\otimes|H\rangle\ + |V\rangle\otimes|V\rangle\ ). \qquad (2.41)$$

The output in Equation (2.41) is one of the *Bell-states*, the commonly used basis in quantum optics consisting of the four maximally entangled states (Exercise 2.10 in the book or kindle version of theQMP) as shown in Equation (2.42):

$$\begin{aligned} |\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle\otimes|\mathrm{V}\rangle + |\mathrm{V}\rangle\otimes|\mathrm{H}\rangle) \\ |\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle\otimes|\mathrm{V}\rangle - |\mathrm{V}\rangle\otimes|\mathrm{H}\rangle) \\ |\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle\otimes|\mathrm{H}\rangle + |\mathrm{V}\rangle\otimes|\mathrm{V}\rangle) \\ |\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle\otimes|\mathrm{H}\rangle - |\mathrm{V}\rangle\otimes|\mathrm{V}\rangle). \end{aligned} \qquad (2.42)$$

Of these four, only the $|\Psi^-\rangle$ is anti-symmetric in interchange of the two photons. For an overall symmetric wave function required by boson statistics, the $|\Psi^-\rangle$ would have to be accompanied by an anti-symmetric spatial wave-function. The other Bell states are symmetric in polarizations and would require a symmetric spatial component. These possibilities for the two-photon statistics have been verified in two-photon interference experiments using polarizing beam splitters [51]. Of particular interest is the behavior called *bunching* which is a consequence of the bosonic statistics and can occur for two photons each incident in two different ports and was found to be maximally entangled in the previous section. These are created by inputting two photons into the beam splitter as in Figure 2.10 (a). If the photons arrive simultaneously, they become indistinguishable and produce a destructive interference called the *Hong-Ou-Mandel* dip in the measured photon coincidence rate [52]. Methods for implementing polarization entangled photons using spontaneous parametric down-conversion (SPDC) are discussed later.