Geometry of the Measurement Problem

Note that state evolution via Schrödinger's equation is a requirement in H_1 in both Tests 3.1 and 3.2. In this respect, first we will examine what occurs during Schrödinger unitary evolution in the interaction of the photon with the particles composing the devices, and later what occurs when a measurement has occurred. Consider an initial photon state in Figure 3.4 that is input into the polarizing beam splitter given by

$$\sqrt{a}|1_V\rangle_A + \sqrt{1-a}|1_H\rangle_A,$$

where $a \in \mathbb{C}$, $|a| \leq 1$. This leads to the initial photon state after the polarizing beam splitter (PBS)

$$|\psi_{\text{photon,PBS}}\rangle = \sqrt{a}|1_V\rangle_B \otimes |0\rangle_C + \sqrt{1-a}|0\rangle_B \otimes |1_H\rangle_C \qquad (3.4)$$

Note that a valid set of Schmidt vectors of the initial photon set can be directly determined via $|e_1\rangle_1 = |1_V\rangle_C$, $|e_2\rangle_1 = |0_H\rangle_B$, $|f_1\rangle_2 = |0_H\rangle_C$, $|f_2\rangle_2 = |1_V\rangle_C$. As all of these vectors are elements of Hilbert space $\mathcal{H}_{\text{photon}}$, with dimension $\mathcal{D}(\mathcal{H}_{\text{photon}}) = 2$, the geometry of this photon state is such that the vectors lie in within a natural basis of Euclidean space as seen in Figure 3.9.

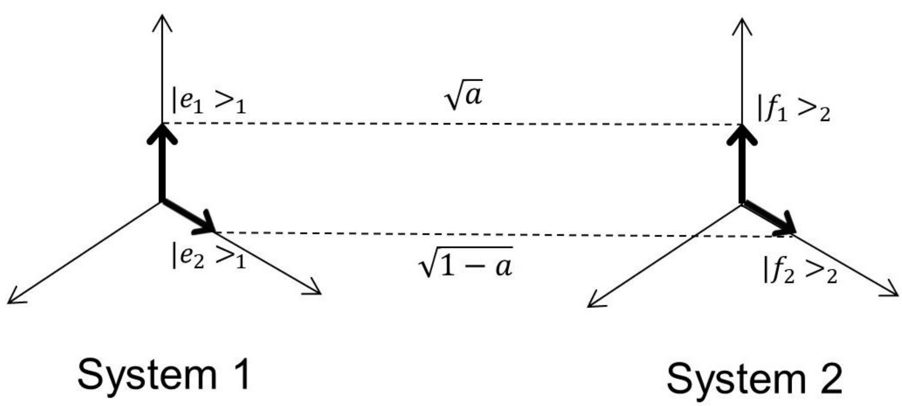

Figure 3.9: Geometry of entangled initial photon.

Combining this with the initial state of the two devices and the two ancilla qubits in Figure 3.4 gives an initial state that will be found to be conveniently ordered as

$$\begin{aligned}|\psi_{\text{init}}\rangle = {} & \sqrt{a}|0\rangle_{A_1} \otimes |1_V\rangle_B \otimes |\psi_r^0\rangle_1 \otimes |0\rangle_{A_2} \otimes |0_H\rangle_C \otimes |\psi_r^0\rangle_2 \\ & + (1-\sqrt{a})|0\rangle_{A_1} \otimes |0_H\rangle_B \otimes |\psi_r^0\rangle_1 \otimes |0\rangle_{A_2} \otimes |1_H\rangle_C \\ & \otimes |\psi_r^0\rangle_2)\end{aligned} \qquad (3.5)$$

where $|\psi_r^0\rangle_i \in \mathcal{H}_i$, i=1,2, $\mathcal{D}(\mathcal{H}_i) = N$, $|0\rangle_{A_1} \in \mathcal{H}_{A_1}$, $|0\rangle_{A2} \in \mathcal{H}_{A2}$, $\mathcal{D}(\mathcal{H}_{A_1}) = \mathcal{D}(\mathcal{H}_{A2}) = 2$.

We have included in $|\psi_{init}\rangle$ the state space of the two ancilla qubits that are depicted in Figure 3.4. Ancilla 1 and the B photon modes have been inserted adjacent to Device 1, while Ancilla 2 and the C photon modes have been inserted adjacent to Device 2. The composite system composed of the state of the photon in the B port and Device 1 will be referred to as System $1'$. System $2'$ denotes the state of the photon in the C port and Device 2. The ancillae are assumed initially uncoupled with Systems $1'$ and $2'$ and remain in the same state during the Schrödinger unitary evolution whereby the photon interacts with the two devices. For notational simplicity, these constant states of the ancillae will not be written but are assumed present, and will be inserted later when the ancillae are directly utilized, resulting in

$$|\psi_{\text{init}}\rangle = \sqrt{a}|e_1^0\rangle_{1'} \otimes |f_1^0\rangle_{2'} + \sqrt{1-a}|e_2^0\rangle_{1'} \otimes |f_2^0\rangle_{2'} \tag{3.6}$$

where

$$\begin{aligned} |e_1^0\rangle_{1'} &= |1_V\rangle_B \otimes |\psi_r^0\rangle_1 \\ |e_2^0\rangle_{1'} &= |0\rangle_B \otimes |\psi_r^0\rangle_1 \\ |f_1^0\rangle_{2'} &= |0\rangle_C \otimes |\psi_r^0\rangle_2 \\ |f_2^0\rangle_{2'} &= |1_H\rangle_C \otimes |\psi_r^0\rangle_2. \end{aligned}$$

The Hamiltonian is assumed to lack interaction terms between Device 1 and 2, likewise there are no photon interaction terms between Arm B and Arm C in Figure 3.4 locally at Device 1 and 2 respectively. It is assumed under Hypothesis H_0 of Hypothesis Test 3.1 that the state evolution follows Schrödinger's equation. Due to the lack of interaction between Systems $1'$ and $2'$, the Hamiltonian consists of interaction terms only between the individual particles composing a device (which is assumed to include all time-like particles within a local environment) and between the individual devices and the respective local arms of the photon. The Schrödinger evolution in Step 1, therefore, takes the form of a tensor product operating on Systems $1'$ and $2'$ with W and V Schrödinger unitary. Hence

$$U = e^{-iHt} = (I \otimes W) \otimes (I \otimes V) \tag{3.7}$$

and $|\psi_{fin}\rangle = (I \otimes W) \otimes (I \otimes V)\, |\psi_{init}\rangle$. The ancillae are multiplied by the identity operator and are unaffected by U. Substituting Equation (3.6), the initial state under Schrödinger unitary evolution evolves to

$$\begin{aligned} |\psi_{\text{fin},1}\rangle &= \sqrt{a}W|e_1^0\rangle_{1'} \otimes V|f_1^0\rangle_{2'} + \sqrt{1-a}W|e_2^0\rangle_{1'} \otimes V|f_2^0\rangle_{2'} \\ |\psi_{\text{fin},1}\rangle &= \sqrt{a}|e_1^1\rangle_{1'} \otimes |f_1^1\rangle_{2'} + \sqrt{1-a}|e_2^1\rangle_{1'} \otimes |f_2^1\rangle_{2'}, \end{aligned} \tag{3.8}$$

where the ancillae have again been dropped from the notation for simplicity,

$$|e_i^1\rangle_{1'} = W|e_i^0\rangle_{1'},\ |f_i^1\rangle_{2'} = V|f_i^0\rangle_{2'},\ i = 1,2.$$

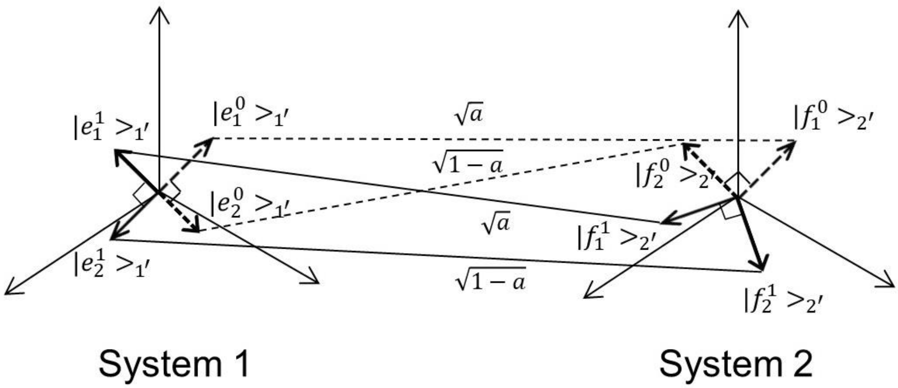

Figure 3.10: Geometry of the Schmidt states in the quantum measurement problem.

The geometry of the initial states and final states of Figure 3.4 is illustrated in Figure 3.10 in terms of the Schmidt decomposition; the ancillae are not affected by the Schrödinger unitary and are not shown. The two orthogonal vectors $\{|e_1^0\rangle_{1'}, |e_2^0\rangle_{1'}\}$ are seen to be rotated into $\{|e_1^1\rangle_{1'}, |e_2^1\rangle_{1'}\}$ and similarly $\{|f_1^0\rangle_{1'}, |f_2^0\rangle_{1'}\}$ are rotated into $\{|f_1^1\rangle_{1'}, |f_2^1\rangle_{1'}\}$. The geometry of orthogonality is maintained as well as the relationships of the respective coefficients in the superposition given by $\sqrt{a}$ and $\sqrt{1-a}$.

One can also examine the geometry of the Schrödinger unitary evolution of Figure 3.4 in a manner similar to Figure 3.8. Shown in Figure 3.11 is the initial state geometry of the superposition in terms of the product states.

The final entangled state under Schrödinger unitary evolution is illustrated in Figure 3.12. A comparison of Figure 3.11 with Figure 3.12 shows that the geometry between the vectors is the same; the two figures are related by rotation. If we assume the devices absorb the photon with probability, one which will be defined as a 100% absorbing device, it follows

$$
\begin{aligned}
|e_1^1\rangle_{1'} &= |0\rangle_B \otimes |\psi^1\rangle_1 \\
|e_2^1\rangle_{1'} &= |0\rangle_B \otimes |\psi^0\rangle_1 \\
|f_1^1\rangle_{2'} &= |0\rangle_C \otimes |\psi^0\rangle_2 \\
|f_2^1\rangle_{2'} &= |0\rangle_C \otimes |\psi^1\rangle_2 .
\end{aligned}
$$

One can then factor out the electromagnetic field in the final Schrödinger predicted state in Step 1, resulting in:

$$|\psi_{\text{fin},1}\rangle = |0\rangle \otimes (\sqrt{a}|\psi^1\rangle_1 \otimes |\psi^0\rangle_2 + \sqrt{1-a}|\psi^0\rangle_1 \otimes |\psi^1\rangle_2. \tag{3.9}$$

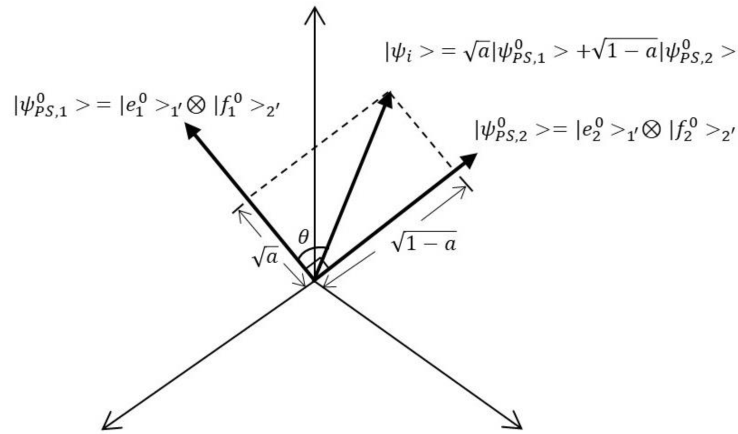

Figure 3.11 Geometry of the initial entangled superposition state composed of two product states in the quantum measurement problem.

We will see from this latter expression that under Schrödinger unitary evolution, the entanglement that originally existed within the photon field has been swapped into entanglement between the initial and final device states. This also aids in explaining why Figure 3.11 and Figure 3.12 are both 2-dimensional planes.

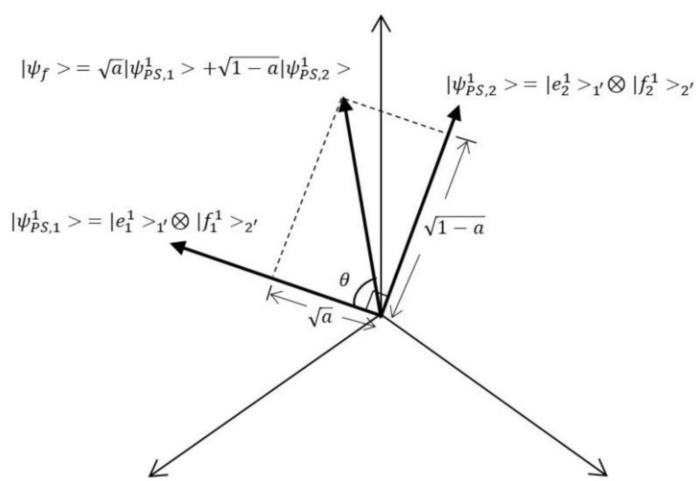

Figure 3.12 Geometry of the final entangled superposition state in the quantum measurement problem.

On the other hand, if the devices do not operate in the sense that a photon was either not absorbed 100% of the time by the device or has been re-emitted with some probability by the device, then the Schrödinger unitarily predicted final state can be an entangled state of field-device. One might consider a solution via post-selection such that only final states are further analyzed when the particular case in which no photon is found in the local electromagnetic field surrounding each device. However, it can be shown (Exercise 3.20 in the book or kindle version of theQMP) that such post-selection can still be problematic in that it is not guaranteed that the final states are orthogonal, i.e. $|\psi^0\rangle_i \perp |\psi^1\rangle_i$ (although there may exist initial states for which this orthogonality does occur). In order to consider the problem for all devices in full generality for arbitrary initial states, and particularly to illustrate the theory for specific device-particle models, each device and its local electromagnetic field as previously defined by Systems $1'$ and $2'$ will be considered. Unitarily it can be shown (Exercise 3.1 in the book or kindle version of theQMP) that these local joint systems do provide the desired orthogonality.