Hypothesis Tests

At this point, one might consider what Bell's inequality has to do with the measurement problem. Although it is generally accepted as a refutation of local hidden variables as regards the action of entangled states, the case for its application to the measurement problem has yet to be justified. In the measurement problem, it is desirable to know whether or not the particles composing a particular device are actually a bona fide detector in the sense of corresponding to the expected observations given by Born's rule. In this case, it is desired to determine if the particles that compose a device are indeed conducting a measurement in a sense that is 1) different than Schrödinger unitary evolution or 2) whether the particles composing the device are acting unitarily and Born's rule is simply a good approximation to the unitarily predicted evolution. There are two related questions to be considered. Firstly, is unitary evolution of devices sufficient to describe the expected results that are seen via Born's rule in the limit that the number of particles composing the device becomes large? Suppose the answer to this first question is found to be that unitary evolution is not sufficient to describe device evolution as an approximation with large numbers of particles. This leads to the second issue which is how to distinguish whether or not a particular device is acting unitarily.

That is, it is desired to utilize Bell's inequality to answer questions that will be put in the form of hypothesis tests:

Hypothesis Test 3.1

H_0: Schrödinger's equation describes the evolution of all particles.
H_1: Schrödinger's equation does not describe the evolution of all particles.

Hypothesis Test 3.2

H_0: Schrödinger unitary evolution of a device as the number of particles increases, is a valid description for the bifurcation rule observed under Born's rule.

H_1: There are cases whereby the quantum state of a device that evolves via Schrödinger unitary evolution does not approach, in the limit of large number of device particles, the expected bifurcation rule observed under Born's rule.

Note that these two hypothesis tests are related but not the same. Consider the possibility in the first hypothesis test that a device is found to evolve non-unitarily. The case of H_0 under Hypothesis Test 3.2, i.e., that Schrödinger unitary evolution with a sufficient number of particles for the description of Born's rule, is not logically ruled out. On the other hand, suppose it is found in Hypothesis Test 3.2 that there are cases whereby Schrödinger unitary evolution of a device with sufficiently large number of particles does not approach the expected observed bifurcation rule observed under

Born's rule. This would either imply that a device is not evolving via Schrödinger's equation in Hypothesis Test 3.1 given the initial assumptions, or the device is evolving according to Schrödinger's equation and the initial assumptions are wrong. That the device is evolving via Schrödinger's equation and the initial assumptions are incorrect will be further considered in Chapter 4.

In considering these questions, the experimental configuration seen in Figure 3.2 is different than in Figure 3.3. In experiments testing Bell's inequality), the source emits two entangled particles, and there is a total of four detectors. On the other hand, in Figure 3.2 there is only a single particle emitted and there are only two detectors. In order to develop the theory in parallel with a Bell-like experiment, the configuration seen in Figure 3.4 will be assumed, whereby two additional ancilla qubits have been added.

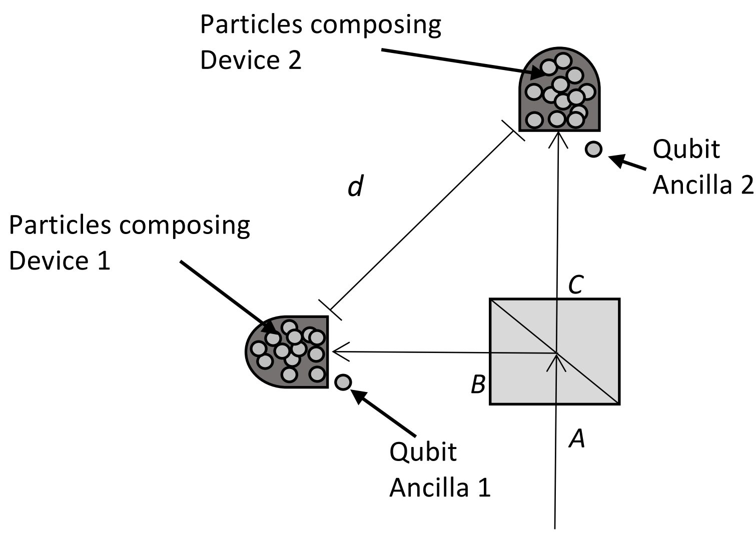

Figure 3.4: Experimental configuration for testing the measurement problem.

The goal is to specify in three steps the operations that will distinguish the Hypotheses:

- Step 1: Provide the Schrödinger unitary evolution that predicts the evolution between a single photon and the two Devices.

- Step 2: A Schrödinger unitary operation is specified that transfers the entanglement that exists between the two Devices (plus a local electromagnetic field if the devices are inefficient) into entanglement between the two ancilla qubits.

- Step 3: Perform a Bell measurement on the two qubits.

It will be seen later that, with the single configuration in Figure 3.4, both Hypothesis Tests 3.1 and 3.2 can be tested experimentally subject to our assumptions.

The secondary measurements are made on the particles that compose the device and on the electromagnetic field locally around the detector. The reason that the secondary measurement includes the electromagnetic field around the detector is that when one considers detectors via simple Hamiltonian models, it may be the case under the hypothesis of Schrödinger evolution that the photon that was absorbed by the device is re-emitted into the electromagnetic field. In order to develop the theory in the most general manner when re-emission is possible, it is required that a secondary measurement is made both on the particles composing the device and on the local electromagnetic field surrounding each device. In this manner, any device Hamiltonian, simple or complex, can be fully considered. On the other hand, if a sufficiently detailed model of a device is used such that the probability of re-emission is small and can be ignored, then it is a good approximation that the electromagnetic field will be found in the vacuum state after a sufficiently long interaction time. In such cases, the electromagnetic vacuum state is the same for all the terms in the analysis and can be factored out, and secondary measurements are only needed on the particles that compose the individual devices.

As can be seen in Figure 3.4, the measurement problem is now cast in a manner similar to Bell's inequality, with the exceptions that 1) the particles that compose the device under question are not *a priori* assumed to be a bona fide detection device, and 2) a secondary measurement takes place on the particles that compose a given device and the local electromagnetic field around each device. The answers to Hypothesis Tests 3.1 or 3.2 are desired. That is, we don't know whether the device, if evolving via Schrödinger's equation, would be a sufficiently good approximation to Born's law for all purposes, nor do we yet know if the device evolves according to Schrödinger's equation or otherwise. Step 1 in our development is to specify a Schrödinger unitary operation on the particles that compose the devices and surrounding electromagnetic field shown in Figure 3.4 in a manner to transfer the entanglement to the two ancilla particles. After that is completed, the last step is to conduct a Bell experiment on the two ancillae.

The analysis to follow, as well as a geometric picture of the Schrödinger unitary evolution, relies on a theorem known as the Schmidt decomposition.