Hamiltonian Description

In Chapter 2, the characteristics of unitary evolution were developed using thought experiments with mirrors and beam splitters. Quantum mechanics employs the Hamiltonian description to describe wave function evolution under Schrödinger's equation. We will assume a spin system initially to represent the detector, although the results will be shown to be generalizable to any system of particles representing the detector.

A beam splitter will be extensively utilized throughout this chapter in the process of developing a unitary versus measurement discrimination test (UMDT). Unless otherwise noted, the beam splitter will be assumed to be a polarizing beam splitter such that the action of the beam splitter depends on the polarization of the input. The states $|1_H\rangle$ and $|1_V\rangle$ denote single photon Fock states of horizontal and vertical polarization respectively, relative to the alignment of the surface of the beam splitter.

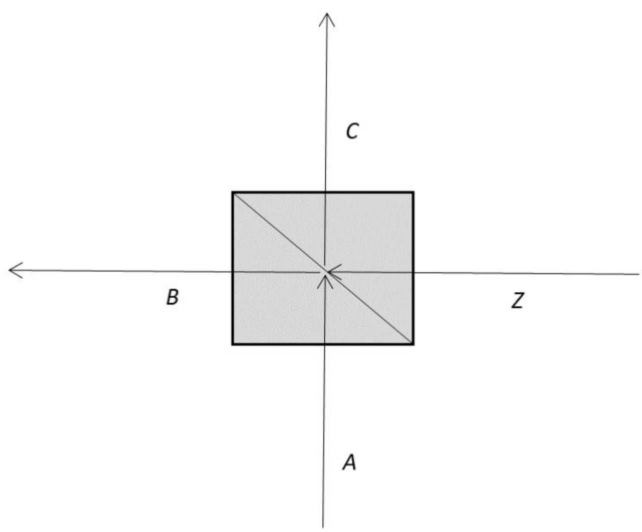

Figure 3.1: 4-port beam splitter, inputs A and Z.

A beam splitter is a four-port device as illustrated in Figure 3.1 for which there are two input ports *A* and *Z*, and in the case of a polarizing beam splitter, two possible polarization states for each port. Unless otherwise noted, it will be assumed that the input to Port Z is the vacuum state, denoted $|0\rangle_Z$ so that any (pure state) input to Port A is a linear combination of the kets $\{|1_H\rangle_A, |1_V\rangle_A\}$. For such a configuration, assuming the beam splitter transmits horizontal polarization and reflects vertical polarization on either Port A or Port Z [80, p. 142], then $|1_H\rangle_A$ evolves to $|1_H\rangle_C$, $|1_V\rangle_A$ evolves to $|1_V\rangle_B$, $|1_H\rangle_Z$ evolves to $|1_H\rangle_B$, and $|1_V\rangle_Z$ evolves to $|1_V\rangle_C$.

Consider a photon that can enter a polarizing beam splitter at Port *A* and for which the two output ports *B* and *C* are directed toward two photon detectors as illustrated in

Figure 3.2. If the state $(|1_H\rangle_A + |1_V\rangle_A)/\sqrt{2}$ is input into the beam splitter, the output is given by $(|0\rangle_B|1_H\rangle_C + |1_V\rangle_B|0\rangle_C)/\sqrt{2}$, or in short-hand dropping the vacuum states this state is often written as $(|1_H\rangle_C + |1_V\rangle_B)/\sqrt{2}$. The same input photon is predicted to exist in a superposition of both output ports.

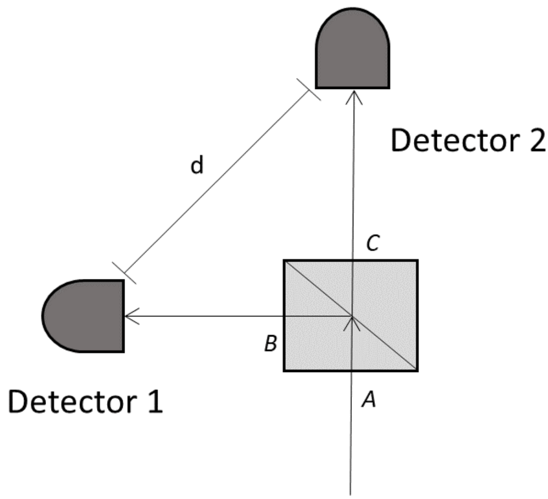

Figure 3.2: Detector configuration for a single photon impinging on beam splitter at A.

Suppose that the detectors are 100% efficient, meaning that if the photon is in the state $|1_V\rangle_C$ then Detector 1 will always register a detection of the photon whereas Detector 2 will never register a detection. Similarly, if the photon is in the state $|1_H\rangle_C$, Detector 2 will always register a detection of the photon whereas Detector 1 will never register a detection. Both Detector 1 and Detector 2 are composed of atomic particles (which might also be in a state of condensed matter) and are subject to a Hamiltonian description.

The photon Hamiltonian (after the polarizing beam splitter) will be assumed to be composed of the two field modes B and C so that $H_F = H_B \otimes H_C$. Detector 1 is assumed to be composed of n_1 spin ½ particles (the arguments given in this chapter are independent of the spin and can be extended to arbitrary spin particles) and has the self-interaction Hamiltonian H_1. Similarly, Detector 2 is composed of n_2 spin ½ particles and has the self-interaction Hamiltonian H_2. Consider the general class of Hamiltonians given by

$$H = H_F \otimes I \otimes I + I \otimes H_1 \otimes I + I \otimes I \otimes H_2 + H_{int} \\ H_{int} = H_{F,1} + H_{F,2}. \quad (3.1)$$

$H_{F,1}$ denotes the interaction Hamiltonian between the photon and Detector 1 while $H_{F,2}$ is the interaction Hamiltonian between the photon and Detector 2. The Hilbert space of the input photon is denoted $\mathcal{H}_A$, similarly $\mathcal{H}_B \otimes \mathcal{H}_C$, $\mathcal{H}_1$, and $\mathcal{H}_2$ denote the Hilbert spaces of the output photon state from the polarizing beam splitter and the

Hilbert spaces of Detector 1 and 2 respectively. Furthermore, $\mathcal{D}(\mathcal{H}_A) = 2, \mathcal{D}(\mathcal{H}_B) = 2, \mathcal{D}(\mathcal{H}_C) = 2, \mathcal{D}(\mathcal{H}_1) = n_1, \mathcal{D}(\mathcal{H}_2) = n_2$, where $\mathcal{D}(\mathcal{H})$ denotes the dimension of the Hilbert space $\mathcal{H}$.