Schmidt Decomposition

The following theorems regarding Schmidt decomposition will be utilized in the analysis to follow.

Theorem 3.1: Consider two quantum systems 1 and 2 with pure states $|\psi\rangle_1 \in \mathcal{H}_1$, $|\chi\rangle_2 \in \mathcal{H}_2$ where $\mathcal{H}_1$ and $\mathcal{H}_2$ denote the Hilbert space of states of systems 1 and 2 respectively. Let $|\psi\rangle$ be a pure state of the composite system 1-2, $|\psi\rangle \in \mathcal{H}_{1,2}$ where $\mathcal{H}_{1,2} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2$. There exist orthonormal states $|e_i\rangle \in \mathcal{H}_1$ and $|f_i\rangle \in \mathcal{H}_2$ whereby

1. $|\psi\rangle = \sum_{i=1}^{N} \sqrt{v_i}\, |e_i\rangle \otimes |f_i\rangle$
2. v_i are non-negative and satisfy $\sum_i v_i = 1$.
3. N is defined as the Schmidt rank.

Theorem 3.2: Suppose $|\psi\rangle \in \mathcal{H}_{1,2}$, $|\phi\rangle \in \mathcal{H}_{1,2}$ are pure states of the joint system 1-2 that have the same Schmidt coefficients. There exist local unitary transformations X, Y acting on systems 1 and 2 respectively such that $|\psi\rangle = X \otimes Y\, |\phi\rangle$.

More information on Schmidt decomposition can be found in [14]. In order to gain further insight into the measurement problem, consider the visualization of the major issues involved.