Specific Device-Particle Modeling

The new UMDT construction applicable to mixed states can now be applied to a specific Device-Particle Model. We will consider the same Hamiltonian of the devices given by Equation (3.14) as in the previous case when the devices were initialized to a pure state. Consider the initial density matrices of the two spins that compose Device 1 and Device 2:

$$\rho_{r,1}^{(0)} = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho_{r,2}^{(0)} = \begin{pmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 3/8 & 3/8 & 0 \\ 0 & 3/8 & 3/8 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Note that $\rho_{r,1}^{(0)}$ is a rank 3 matrix with a repeated eigenvalue of ¼ and can be shown to be an entangled two-spin state. There is a 2-dimensional subspace that is spanned by two orthogonal eigenstates corresponding to the eigenvalue 1/4. Consider the following choice of orthogonal eigenstates of $\rho_{r,1}^{(0)}$:

$$|\psi_{r,1}^{0}\rangle_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\psi_{r,2}^{0}\rangle_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |\psi_{r,3}^{0}\rangle_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$\rho_{r,2}^{(0)}$ is a rank 2 matrix with no repeated eigenvalues. Consider the following choice of orthogonal eigenstates of $\rho_{r,2}^{(0)}$:

$$|\psi_{r,1}^{0}\rangle_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\psi_{r,2}^{0}\rangle_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Now construct for both $j = 1,2$

$$\begin{aligned} |e_{1,j,}^{0}\rangle_{1'} &= |1_V\rangle_B \otimes |\psi_{r,j}^{0}\rangle_1 \\ |e_{2,j}^{0}\rangle_{1'} &= |0\rangle_B \otimes |\psi_{r,j}^{0}\rangle_1 \\ |f_{1,j}^{0}\rangle_{2'} &= |0\rangle_C \otimes |\psi_{r,j}^{0}\rangle_2 \\ |f_{2,j}^{0}\rangle_{2'} &= |1_H\rangle_C \otimes \left|\psi_{r,j}^{0}\right\rangle_2 \end{aligned}$$

which are unitarily evolved via Schrödinger's equation in Step 1 to $\left|e_{i,j}^{1}\right\rangle_{1'} = W\left|e_{i,j}^{0}\right\rangle_{1'}$ and $\left|f_{i,j}^{1}\right\rangle_{2'} = V\left|f_{i,j}^{0}\right\rangle_{2'}$, for $i = 1,2, j = 1,2$. In order to specify the Step 2 operation, consider the composite Ancilla 1-System $1'$. The two matrices $T_{1,1}$ and $T_{1,2}$ are computed via

$$T_{1,j} = (|1\rangle_{A_1} \otimes |e^1_{1,j}\rangle_{1'})(\langle 0|_{A_1} \otimes {}_{1'}\langle e^1_{1,j}|) + (|0\rangle_{A_1} \otimes |e^1_{1,j}\rangle_{1'})(\langle 0|_{A_1} \otimes {}_{1'}\langle e^1_{2,j}|)$$

and the sum is formed, $T_1 = T_{1,1} + T_{1,2}$, which can be extended to a unitary operation X with use of the singular value decomposition (Exercises 3.10-3.12 in the book or kindle version of theQMP). Utilizing the same procedure applied to the composite Ancilla 2-System $2'$, a unitary operation Y is found and $X \otimes Y$ is applied to complete Step 2.

Once Step 2 is completed, the ancilla qubits are extracted from the overall state and a Bell (CHSH) experiment is performed. The computation for the initial state above was found to result via computer simulation in Figure 3.17 for the Hamiltonian with parameters considered previously, i.e. $t = .4$ using the Hamiltonian parameters $w = 2$, $\{\Omega_{1,1,}\Omega_{2,1,,}J_{x,1}, J_{y,1}, J_{z,1}\} = \{.5, .2, .1, .3, .5\}$ and $\{\Omega_{1,2,}\Omega_{2,2,,}J_{x,2}, J_{y,2}, J_{z,2}\} = \{.3, .4, .7, .2, .6\}$. As expected from the mixed-state two-qubit CHSH sum result in Equation (3.28), it can be seen that this result in Figure 3.17 is identical to Figure 3.16 which is the case when the devices are initialized to pure states. Moreover, the unitary operation that is utilized in Step 2 is not of the form that requires a time-reversal of the unitary that occurred in Step 1.

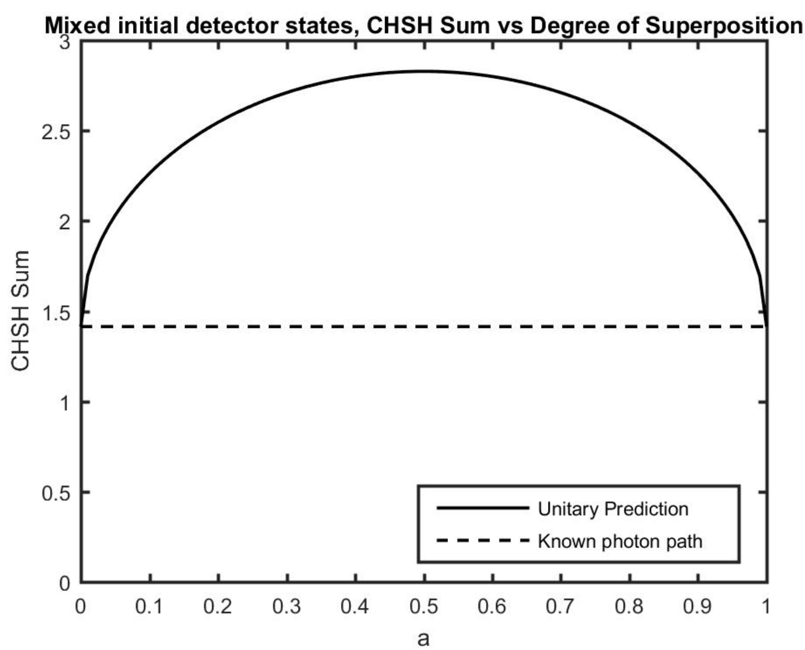

Figure 3.17 CHSH Sum versus degree of photon superposition when initial device states are mixed.