Specification of Operations

Equation (3.8) will be the starting point for the Step 2 operation. In Step 2 local unitary operations will be applied that evolve Equation (3.8) to a state for which the entanglement that was present between the two devices is swapped to create entanglement between the two ancilla qubits. Step 3 is to perform a Bell measurement on the two qubits. At this point, the two ancilla qubits are included in the description of $|\psi_{\text{fin}}\rangle$ resulting in,

$$|\psi_{fin,1}\rangle = \sqrt{a}|0\rangle_{A_1} \otimes |e_1^1\rangle_{1'} \otimes |0\rangle_{A_2} \otimes |f_1^1\rangle_{2'} + \sqrt{1-a}|0\rangle_{A_1} \otimes |e_2^1\rangle_{1'} \otimes |0\rangle_{A_2} \otimes |f_2^1\rangle_{2'}. \tag{3.10}$$

The operations that are desired in Figure 3.4 are such that they will enable the discrimination between the hypotheses in Tests 3.1 and 3.2. Firstly, we will develop the procedure needed for addressing Test 3.2; it will be seen later that the same procedure also suffices to enable the discrimination of the hypotheses in Test 3.1.

A reasonable question is whether or not the entanglement that is unitarily predicted between System $1'$ and $2'$ can necessarily be transferred to the qubits in Step 2. Note that as $U = (I \otimes W) \otimes (I \otimes V)$ in Step 1, one could simply choose the Step 2 operation to be $U^{-1} = (I \otimes W^{-1}) \otimes (I \otimes V^{-1})$, which reverses the original interaction and transfers the entanglement back to the photon field. Continue by interacting the local fields B and C in a manner that transfers the entanglement between the single photon and the qubits similar to the approach taken in [101]. This shows that it is always possible to find a local Step 2 operation, by reversing the original interaction and transferring the photon superposition into two-qubit entanglement.

However, there are several reasons to consider if it is possible to transfer the entanglement without reversing the original detector-field unitary operation. For example, one might object to such inverse methodology as being at the very least extremely difficult as it requires the implementation of a new Hamiltonian that accomplishes the complete time-reversal of the naturally occurring process of the original interaction. Another important reason is that it is desirable to work towards implementable techniques that can discriminate unitary evolution from measurement. As well, there are theories that will be examined in Chapter 4 that maintain that the measurement problem is resolved by certain naturally occurring unitary processes that ultimately cannot be reversed. With these reasons in mind, we will not allow the consideration of Step 2 unitary operations that require the time-reversal of the detector. If there are terms in the device state after Step 1 for which the device has evolved from an initial state to a final state (or in the case of a mixed initial device state from an initial eigenstate of the device to a final state), the only operations that will be considered allow the device to remain in such states or evolve further in time via the original unitary.

The final vectors in Figure 3.12 lie in a 2-dimensional plane. These vectors are typically in a high dimensional subspace determined by the particles that compose the detectors. We begin by showing there exists a local unitary matrix that rotates these vectors into any two natural Euclidean basis vectors, as any two such basis vectors also form a two-dimensional space, which can contain the plane of the final vectors. This operation will be implemented between the particles that compose each device and the respective local qubit, as shown in Figure 3.13.

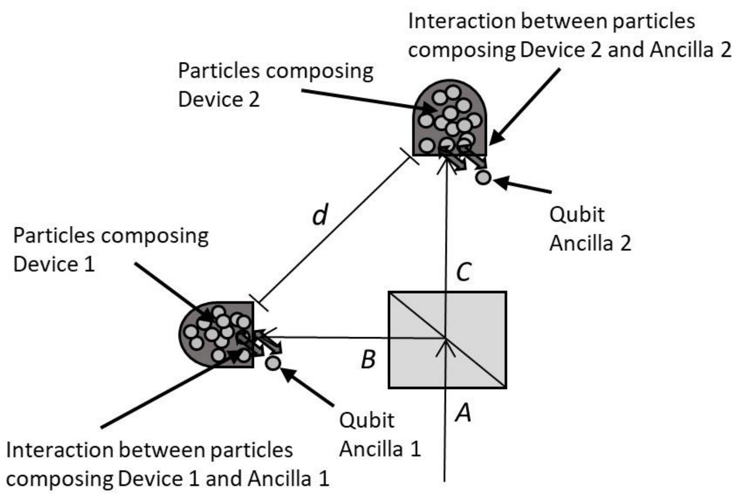

Figure 3.13 Operation of interaction between the particle composing each device and the respective ancilla qubits and local electromagnetic field.

Define

$$
\begin{aligned}
T_1 &= (|1\rangle_{A_1} \otimes |e_1^1\rangle_{1'})(\langle 0|_{A_1} \otimes {}_{1'}\langle e_1^1|) + (|0\rangle_{A_1} \otimes |e_1^1\rangle_{1'})(\langle 0|_{A_1} \otimes {}_{1'}\langle e_2^1|) \\
T_2 &= (|1\rangle_{A_2} \otimes |f_2^1\rangle_{2'})(\langle 0|_{A_2} \otimes_{2'} \langle f_2^1|) - (|0\rangle_{A_2} \otimes |f_2^1)\rangle_{2'})(\langle 0|_{A_2} \otimes {}_{2'}\langle f_1^1|)
\end{aligned} \tag{3.11}
$$

where $|0\rangle_{A1}$, $|1\rangle_{A1}$ are natural basis vectors of the ancilla qubit (a second ancilla qubit is similarly required for System $2'$),

$$|0\rangle_{A_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle_{A2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

As it was assumed that both System 1 and 2 have dimension N, T_1 and T_2 are $2N$ by $2N$ matrices with 2 non-zero rows. T_1 will have the effect of mapping $|0\rangle_{A_1} \otimes |e_1^1\rangle_{1'}$ to $|1\rangle_{A_1} \otimes |e_1^1\rangle_{1'}$ and $|0\rangle_{A_1} \otimes |e_2^1\rangle_{1'}$ to $|0\rangle_{A_1} \otimes |e_1^1\rangle_{1'}$, where $|1\rangle_1 \in \mathcal{H}_1$. Note that this operation has similar functionality as a controlled-not operation commonly encountered in quantum computing, although applied to a device rather than a qubit.

Consider the specification of a unitary matrix X as an extension of T_1 and Y as an extension of T_2 (the X and Y extensions do not need to be identical). Let X have the same two non-zero rows as T_1 and Y the same two non-zero rows as T_2. The rows of X

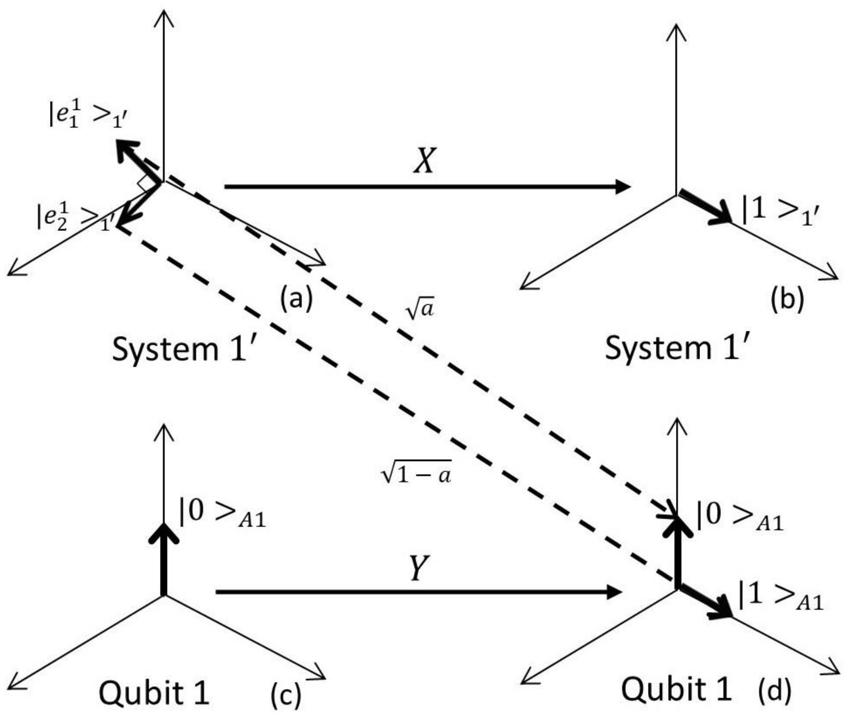

Figure 3.14: The unitarily evolved Schmidt vectors in Systems $1'$ are shown in Figures (a), (b) operated on by X causing the initial ancilla qubit 1 in Figure (c) to evolve to Figure (d) in a manner that retains the coefficients $\sqrt{a}, \sqrt{1-a}$ of the initial state of System$1'$ in Figure (a). A similar diagram is found for System $2'$ via the transformation Y_t.

will be orthonormal if and only if $|e_1^1\rangle_{1'}$ is orthogonal to $|e_1^2\rangle_{1'}$. Note from the previous discussion regarding efficient detection, such orthogonality is a reasonable assumption for the device state of many realistic detectors; however, there exist models for which orthogonality is not guaranteed, which is why we extended Systems 1 and 2 to Systems $1'$ and $2'$ respectively. Assuming these rows are orthonormal, one can complete X (and similarly Y) by adding $2N-2$ rows such that all rows form a complete basis of the $2N$ dimensional linear vector space. As the rows form a complete orthonormal basis and the matrices are square, X and Y are also unitary. Note that we chose to map both $|e_1^1\rangle_{1'}$ and $|e_1^2\rangle_{1'}$ to $|e_1^1\rangle_{1'}$ in the Hilbert space $\mathcal{H}_1$. However, it can be shown for the case when the initial detector state is pure (Exercise 3.3 in the book or kindle version of theQMP) that an arbitrary state in $\mathcal{H}_1$ could have been chosen (and similarly for System 2). Conditions that the chosen final states must meet when the initial detector state is mixed are examined in Exercise 3.14 in the book or kindle version of theQMP.

Let X be such a $2N$ by $2N$ extension of T_1 and Y an extension of T_2 (the X and Y extensions do not need to be identical). The geometrical effect of X is to transfer the quantum state coefficients of the vectors $\{|e_1^1\rangle, |e_2^1\rangle\}$ into the coefficients of the natural vectors of the ancilla qubit A_1, as seen in Figure 3.14. Step 2 is to apply $X \otimes Y$ to Equation (3.10) resulting in

$$|\psi_{fin,2}\rangle \equiv (X \otimes Y)\, |\psi_{fin,1}\rangle \tag{3.12}$$

$$|\psi_{\text{fin},2}\rangle = \sqrt{a}|1\rangle_{A_1} \otimes |e_1^1\rangle_{1'} \otimes |0\rangle_{A_2} \otimes |f_2^1\rangle_{2'} + \sqrt{1-a}|0\rangle_{A_1} \otimes |e_1^1\rangle_{1'} \otimes |1\rangle_{A_2} \otimes |f_2^1\rangle_{2'}\,.$$

This can be rewritten by swapping the second and third systems:

$$= \sqrt{a}|1\rangle_{A_1} \otimes |0\rangle_{A_2} \otimes |e_1^1\rangle_{1'} \otimes |f_2^1\rangle_{2'} + \sqrt{1-a}|0\rangle_{A_1} \otimes |1\rangle_{A_2} \otimes |e_1^1\rangle_{1'} \otimes |f_2^1\rangle_{2'}$$

for which the state of the detector particles can be factored out:

$$= \{\sqrt{a}|1\rangle_{A_1} \otimes |0\rangle_{A_2} - \sqrt{1-a}|0\rangle_{A_1} \otimes |1\rangle_{A_2}\} \otimes |e_1^1\rangle_{1'} \otimes |f_2^1\rangle_{2'}, \tag{3.13}$$

which includes our desired 2-qubit entangled state (in the first two systems).

Now that Step 2 has been completed, a standard Bell experiment is required to be implemented on the two ancillae for completion of Step 3 (utilized in the quantification of the Bell inequality), as shown in Figure 3.15.

Note that the unitary operations that are utilized in Step 2 are not of the form of the time-reversal of the unitary that occurred in Step 1. Hence it has been demonstrated that the entanglement can be unitarily transferred without the impediment of requiring a large-scale time-reversal, which was a desired property of the operation for the reasons discussed previously. We will refer to the use of Figure 3.4 along with device Hamiltonian and the Step 1, 2, and 3 operations that have now been fully specified as a *unitary versus measurement discrimination test* (UMDT).