Bohm's Theory

Bohm's theory is one of the several early attempts to come to grips with the measurement postulate that fall within the class of no-collapse theories in which it is desired to explain measurement without use of the measurement "collapse" postulate. In Bohm's theory, the Schrödinger wave function does not represent the particles' actual trajectory, but rather all possible trajectories that could occur. Bohm uses the Schrödinger wave function to derive a quantum potential which acts on and guides the particle with a force and for which the particle's velocity can be derived.

Bohm's theory is unique in that it includes aspects of Schrödinger's equation and also incorporates a spatially localized trajectory that the particle is said to follow. Consider Schrödinger's equation given by

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi. \tag{4.16}$$

The wave function is generally complex and can be written as $\psi = Re^{iS/\hbar}$, where R is a real time varying function. The probability of measurement under Born's interpretation is $|\psi|^2$ which is a function of R. Substituting $\psi = Re^{iS/\hbar}$ into Equation (4.16) and taking the real part of both sides yields

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + \frac{\hbar}{2m}\frac{\nabla^2 R}{R}. \tag{4.17}$$

When $\hbar = 0$ this is a classical Hamilton-Jacobi equation where S is the action. If one considers a modified potential

$$\bar{V} = V + \frac{\hbar}{2m}\frac{\nabla^2 R}{R}$$

then a solution to Equation (4.17) would also constitute a solution to the original Schrödinger equation. In such a case, the velocity of the particle can be derived from the Hamilton-Jacobi equation as $v(x,t) = \nabla S(x,t)/m$. The right-hand side of Equation (4.17) is only a function of the action S and the potential V. Equation (4.17) can also be interpreted as a modified Hamiltonian-Jacobi equation where S is the action resulting from a classical potential V and a newly added quantum potential Q

$$Q = -\frac{\hbar}{2m}\frac{\nabla^2 R}{R}. \tag{4.18}$$

Note that the quantum potential is a function of R which is related to the Born probability of measurement. This is a rather interesting result by Bohm and at this stage indeed shows promise for resolving the measurement problem. One can derive from the quantum potential Q a force $F = -\nabla Q$ for which the particles are guided by the quantum potential which is in addition to the force on the particle by the potential V. The particle will have a well-defined position at all times but will experience a force as a function of the quantum potential. The quantum potential is in general non-local. Bohm's theory is an example of a non-local hidden variables theory.

This theory is of the type where you can "have your cake and eat it too." Particles travel in a classical manner yet are guided by a non-classical wave function so that the results of measurement would seem to occur in a natural manner. Suppose a theory exists such as Bohm's theory for which particles always take particular trajectories that interact with one and only one detector. By imposing a quantum potential that now guides particles in a local manner, one might expect that the measurement problem can be resolved. It is indeed true that the particles under Bohm's theory can be thought of as guided by a force that is a function of the potential V and also R in a manner that the particle obeys Born's rule. The particles require an initial position as if chosen from the probabilistic density function $|\psi|^2$ and also requires a probabilistic initial velocity that is a function of the wave function and current density. This then might appear to be a methodology for which the measurement postulate is not actually needed as the particles already are taking classical paths. In this case some incorrectly conclude that, simply because such a theory exists, the measurement problem can be relegated to an interpretational problem and is not a problem that could have any significant consequences upon its solution.

The main objective of this chapter is for the reader to further learn how to discern correct approaches from incorrect approaches regarding the measurement problem. The devil is in the details, and because of the subtleties involved, Bohm's theory presents an excellent opportunity to analyze. With this in mind, it is interesting to note that in the 1984 paper by Bohm [151], in a section entitled "The Quantum Mechanical Process of Measurement" where he considers the measurement problem, Bohm divides the measurement process into two stages.

See the print edition of The Quantum Measurement Problem for quotation.

Bohm [151] gives an example of an in-principle reversible interaction, as the interaction of the angular momentum of an electron in an inhomogeneous magnetic field. That is, the electron might be in a superposition of different spin states for which the paths through the magnetic field would separate the electron. Bohm states that the packets of the electron can fully separate. However, Bohm does agree there is a problem at this point to apply the measurement postulate because it is possible to recombine the packets together. He states, "It is therefore clear that we have not yet explained the irrevocability of the experimental results…" Bohm goes on to explain that it is only in the second stage for which there is an irreversible registration that there is a result of measurement. That the new quantum state can be said to occur, "when the reversal is overwhelming improbable." Bohm goes on to justify this on the basis of irreversibility found in macroscopic systems via thermodynamics. Bohm adds a further requirement to his theory: he requires at this point that when the

> *... quantum potential of the whole system is calculated, the apparatus wave function alone will guarantee that the packet not corresponding*

> *to the actual result can be left out of the discussion. Clearly it is the irreversibility of the process of registration that further guarantees that this conclusion will continue to be valid in the future.... We are thus led to the conclusion that the packets not containing this particle now correspond to inactive information. This is essentially what happens in classical solutions involving probability distributions. In such situations, after a new observation is made, we simply discard the previous probability function.*

One might agree that irreversibility is related to the solution to the measurement problem as Bohm suggests, but this is not what modern papers concentrate on when utilizing Bohm's theory to explain measurement. On the contrary, modern authors generally focus mostly on the theory without the two-step process as a resolution of the measurement problem. In fact, the two-step process that Bohm proposes is either left out entirely of the discussion by the proponents of Bohm's theory for resolving the measurement problem, or it is buried in a few lines in a manner that few would have suspected could play such a huge role. Consider Holland [5, p. 349] in which the two-stage process by Bohm is considered:

> *The complexity of the interaction gives it the appearance of being irreversible but the underlying dynamical law bringing about the wave function remains the reversible Schrödinger equation. That the process so described is not actually irreversible may be considered a weak point in our demonstration.*

Using a reversible process to demonstrate an irreversible process does appear to be a rather weak point. Holland isn't finished however. Holland makes the statement regarding the Stage 2 requirement of Bohm:

> *As we have presented it, the theory of measurement may be understood without first answering fundamental questions regarding the nature of irreversibility.*

If Holland's statement is correct, one might conclude that the measurement problem is also resolved by Bohm's theory without the Stage 2 requirement of Bohm. The latter statement by Holland refers to, "the theory of measurement." The current von Neumann theory of measurement is an *a posteriori* theory of measurement, i.e., given that a measurement has been made, the results must agree with Born's rule. In the sense that Bohm's theory is an *a posteriori* theory, it does indeed reproduce the results of measurement and resolves the philosophers' measurement problem. However, it does not yet meet the requirements of resolving the physical measurement problem.

In addition to Holland there are others that believe that Bohm's theory resolves the measurement problem [152] [153] [114] [154]. Dürr et al. states [155]:

See the print edition of The Quantum Measurement Problem for quotation.

Holland’s and Dürr’s statements are correct in the sense that Bohm’s theory provides a deterministic model that is consistent with the results of the measurement postulate *given* that a measurement has occurred. Hence, they can rest knowing that they have a solution to the philosophers’ measurement problem. But as has been stated, von Neumann quantum theory is an *a posteriori* theory and the measurement problem as we define it, requires one to provide a scientifically validated theory that explains the dichotomy between the entanglement predictions of Schrödinger’s equation versus the product state prediction *without* pre-assuming that measurement has occurred. While it is true that Bohm’s theory is a solution to the philosophical measurement problem, at best it can be considered only one out of many approaches regarding the physical measurement problem. Bohm’s theory does not come close to having been scientifically validated to meet the requirements that we have set in resolving the physical measurement problem.

Historically one might consider that there are two “measurement problems.” Whereupon Schrödinger developed his new equation in 1926 and was invited to Bohr’s home, Bohr argued with Schrödinger that a deterministic equation would not suffice and that the quantum of action of Planck demanded a non-causal or nondeterministic element to explain various phenomena. Born’s probabilistic rule was developed shortly thereafter to explain the results of measurement. This satisfied Bohr and became part of the Copenhagen interpretation. Let us suppose that historically after Born’s rule was proposed, that Schrödinger had available Bohm’s theory. Then Schrödinger could argue with Bohr that Bohm’s theory without irreversibility does reproduce the results of measurement in a deterministic manner and that Born’s rule is only needed due to the ignorance of unknown initial conditions. Bohmian mechanics is a deterministic counterexample that shows Bohmian mechanics reproduces both unitary evolution and the measurement postulate, for which the measurement postulate is a formulation of what to do given that a measurement has occurred. If one defines the measurement problem in a manner that can be resolved via a theory that demonstrates in a deterministic manner Born’s rule *a posteriori* given that a measurement occurs while maintaining unitary evolution, then Bohmians can go to sleep at night content they have solved the measurement problem. And it has been said that ignorance is bliss. But it also has been said that bliss is ignorance.

In 1935, Schrödinger published two important papers [156] [119]. Schrödinger states in [119]

> *As soon as the systems begin to influence each other, the combined function ceases to be a product and moreover does not again divide up, after they have again become separated, into factors that can be assigned individually to the systems. Thus one disposes provisionally (until the entanglement is resolved by an actual observation of) of only a common description of the two in that space of higher dimension.*

Schrödinger is acknowledging that his equation predicts that an initial product state of photon and a system become entangled and cannot be represented as a product state after separating. Schrödinger in the same paper also provides his famous example of such an entangled superposition even if one were to include a living cat as one of the systems. Furthermore, the phrase "is provisional until resolved by an actual observation," is a statement that the entanglement is resolved only when a measurement occurs. The physical measurement problem, as we have defined it, is consistent with this latter issue, that is determining the conditions under which measurement occurs for which the Schrödinger entangled wave function is formally replaced by a product state. The physical measurement problem is not trivially resolved via assumption that a measurement has already occurred; rather we demand that a solution to the measurement problem provide the conditions under which a measurement occurs and the theoretical basis for such conditions.

Bohm's theory does reproduce Born's rule as an *a posteriori* theory. But let us now analyze Bohm's theory without irreversibility to see if does resolve the measurement problem, which requires one to dig deeper without skirting major issues. That Bohmian mechanics has little to offer in the sense that we have defined the measurement problem can be seen when Bohmians explain when they will change the wave function into a new wave function that is indicative of a measurement.

In this regard, consider the treatment of the measurement process by Bohm's theory in Orioles and Mompart [153, pp. 95-99]. The system is specified by a trajectory $\vec{x}_S$ and the apparatus by trajectory $\vec{x}_A$. The possible outcomes of the measurement process correspond to one of the possible eigenvalues g of a Hermitian operator $\hat{G}$ that satisfy the equation $\hat{G}\psi_g(\vec{x}_S) = g\psi_g(\vec{x}_S)$. The system wave function can be decomposed via the eigenstates of $\hat{G}$,

$$\psi_S(\vec{x}_S, t) = \sum_g c_g(t)\psi_g(\vec{x}_S).$$

When measuring the eigenvalue g_a the wave function $\psi_S(\vec{x}_S, t)$ needs to collapse to $\psi_{g_a}(\vec{x}_S, t)$. In addition to the entire apparatus, the pointer of the apparatus is represented by $\vec{x}_P(t)$. When the quantum system is in an eigenstate $\psi_g(\vec{x}_S)$, the pointer is in a region S_g, i.e. $\vec{x}_P(t) \in S_g$. A requirement is added that the pointer positions $\vec{x}_P(t)$ of the apparatus are disjoint in the sense that if g_1 and g_2 are different eigenstates of $\hat{G}$ then the pointer state trajectories are disjoint, i.e. $S_{g_1} \cap S_{g_2} = \emptyset$. A total wave function $\Phi_T(\vec{x}_S, \vec{x}_A, t)$ can then be defined that is decomposed over the possible outcomes of the experiment via the eigenvalues of $\hat{G}$,

$$\Phi_T(\vec{x}_S, \vec{x}_A, t) = \sum_g c_g(t)\, f_g(\vec{x}_A, t)\, \psi_g(\vec{x}_S). \tag{4.19}$$

with the property that $f_{g_1}(\vec{x}_A, t) \cap f_{g_2}(\vec{x}_A, t) = 0$. Now, the Bohmian process of measurement is as follows. An initial trajectory is chosen $\{\vec{x}_S(0), \vec{x}_A(0)\}$

commensurate with a probabilistic outcome of the initial wave function positions of the system and apparatus. Such a trajectory will evolve with the total wave function, and during the measurement, the total trajectory $\{\vec{x}_S(t), \vec{x}_A(t)\}$ will correspond to only a single term of Equation (4.19) which corresponds to $g = g_a$. It is stated [153, p. 97]

> *Thus, the pointer positions will be situated in $\vec{x}_P[t] \in S_{g_a}$ and we will conclude with certainty that the eigenvalue of the quantum system is g_a. In addition, the subsequent evolution of this trajectory can be computed from $f_{g_a}(\vec{x}_A, t)\psi_{g_a}(\vec{x}_S)$ alone. In other words, we do not need the entire wave function $\sum_g c_g(t)\, f_g(\vec{x}_A, t)\, \psi_g(\vec{x}_S)$ because the particle velocity can be computed from $f_{g_a}(\vec{x}_A, t)\psi_{g_a}(\vec{x}_S)$. The rest … are empty waves that do not overlap with $f_{g_a}(\vec{x}_A, t)\psi_{g_a}(\vec{x}_S)$ so that they have no effect on the velocity of the Bohmian particle. This is the simple explanation of how the complicated orthodox collapse is interpreted….*

So far so good! If we could simply stop here, the authors would have indeed resolved the physical measurement problem. However, one cannot stop here and the authors must add an additional requirement [153, p. 98]

> *Finally, we want to enlarge the explanation of the role played by the empty waves belonging to the wave packet of Equation [(4.19) in this text] which do not contain the particle. In principle, one can argue that such empty waves can evolve and, in later times, overlap with the original wave that contains the particle. If we are interested in doing subsequent (i.e., two times) quantum measurements, a good measuring apparatus has to avoid these spurious overlaps. This can be understood as an additional condition for qualifying our measuring apparatus as a good apparatus.*

This statement appears designed to enforce Bohm's Stage 2 irreversibility. Bohm states in [151]:

> *Clearly it is the irreversibility of the process of registration that further guarantees that this conclusion will continue to be valid in the future.*

That is, the packets that do not contain the particle will be guaranteed to remain empty packets as long as the process of registration is irreversible. Let us consider this additional condition in the context of the UMDT of Chapter 3, which was specifically designed to determine what a given measurement theory does and does not accomplish. For simplicity, we assume that the devices are 100% absorbing in the sense defined in Chapter 3. Suppose that the photon after interaction with Device 1 and 2 in Step 1 are in states such that the state for which no measurement occurs does not overlap with the state for which measurement occurs. There are two cases to consider after Step 1. In Case 1 the Devices are bona fide measurement devices in which case the evolved Device 1 state and Device 2 state are in a product state. In Case 2 the interaction between the photon and devices is unitary in which case the final state of Device 1 and Device 2 are in an entangled state. Step 2 is then employed for which the two qubits will remain unentangled under Case 1 and in Case 2 the entanglement in the Devices will be transferred to the two qubits. In Step 3, a Bell experiment is conducted and the CHSH sum is computed. In Case 1, the CHSH will give a value of $\sqrt{2}$ and in Case 2 the CHSH will give a value of $2\sqrt{2}$. The additional Orioles-Mompart condition would imply that a bona fide measurement has only occurred when there is never subsequent overlap between the cases of absorption and non-absorption. If the angles of the Bell experiment were chosen that were 0 degrees or 180 degrees such that the results could be inferred with probability unity from the non-overlapped decomposition after Step 1, then there would be no repercussions in performing the Bell experiment and Bohm's theory will suffice to explain Steps 2 and 3. However, the angles chosen in the Bell experiment that is required in Step 3 are such that the Bell measurement do indeed mix or overlap the empty waves that existed after Step 1. Hence in the case of Bohm's theory with the additional Orioles-Mompart condition designed to enforce irreversibility, Step 3 will always be predicted to yield a CHSH sum of $2\sqrt{2}$ and thus the states would still be entangled.

Suppose that one of the other quantum measurement theories is found to be correct, for example the theory of spontaneous localization. And that such localization occurs after some amount of interaction in the first of the two-time measurements. If so, then the actual quantum state is a product state after the first of the two-time measurements. And after the particles undergo Step 2 and Step 3, the CHSH sum would be $\sqrt{2}$. So, a theory that specifies physical conditions under which measurement occurs such as a spontaneous localization theory, if experimentally found to be correct, would falsify Bohm's theory as stated by Orioles-Mompart which demands that entanglement continues to persist until such time as there are no further overlaps in the various superposition terms. Bohm's theory is predicated on unitary evolution and it is possible that a non-unitary theory that makes specific predictions and meets the requirements R1.1-R1.3 can falsify Bohm's theory, but only if such a theory is ultimately found to be correct.

One might argue that the additional Orioles-Mompart condition can be dropped or is superfluous. However, Bohm in his paper considered a Stern-Gerlach experiment where a spin ½ particle enters an inhomogeneous magnetic field which separates the particle into two distinct paths that the spin up state takes versus the spin down state. Bohm argues that this separation cannot be considered to be a bona fide measurement because the two paths could be further recombined into a single path. If the separation into two paths was a sufficient condition for measurement, then the coherence in the particle would be destroyed by an inhomogeneous magnet and the recombination into a single path would be non-coherent. It has been demonstrated experimentally that such recombination does maintain coherence. Because of this and other counterexamples, Bohm required the second stage of irreversibility.

Let us examine how well the additional Orioles-Mompart condition addresses the physical conditions under which irreversibility occurs. Suppose an experiment were done that separated the electron via an inhomogeneous magnetic field and for which the paths were never recombined. Then one could apply the additional Orioles-Mompart condition and conclude that such a device is a bona fide measurement device. In the case that the electron is separated and then recombined, the additional Orioles-Mompart condition implies the initial separation is not a measurement, but in the second case when the electron is separated and not recombined, the initial separation is a measurement. Hence such a condition classifies irreversibility based on what happens in the future.

The additional Orioles-Mompart requirement is rather constraining and renders a lack of unique predictive power to explain when a particular device is a bona fide measurement device based on the current and past state of the device. For example, instead of considering the electron in an inhomogeneous magnetic field, we consider the interaction of a particle with a mesoscopic device. Suppose a state results that can be written in the form in Equation (4.19). If after the initial interaction one does not allow the empty waves to overlap, then Orioles-Mompart would declare the device to be a “good” apparatus. On the other hand, suppose that after the interaction with a device, further interaction is made to occur. In such a case, Orioles-Mompart would declare this not to be a good apparatus. Hence the ability of Bohm’s theory with the additional Orioles-Mompart requirement to predict whether or not an apparatus is good, is not based on the history of the state nor the current state of the device but must include the future evolution. Hence a device in one situation would be classified a good apparatus and in another would not be, depending on whether or not the empty waves further interact in the future. However, this is rather contrary to what one might expect based on causal theories. That is under a causal theory whether an apparatus is a measurement device or is not should be based on the past state and the present state; what will happen in the future would not be expected to have bearing on this issue.

Bohm’s theory with the added Orioles-Mompart condition is non-unique in the manner that it classifies good measurement devices based on full knowledge of the current and past data, which is typically demanded of any physical theory. The additional Orioles-Mompart condition does not provide predictability to explain in the sense of our requirement R1.1, “What physical situations constitute the divide of irreversibility?” Essentially, we know that it is somewhat more complex than a single electron going through an inhomogeneous magnetic field and perhaps on the order of a macroscopic system consisting of $\approx 10^{23}$ particles. There is no precise criterion in Bohm’s theory that provides a prediction of the onset of irreversibility.

Bohm’s theory may resolve the philosophers’ measurement problem as stated by Lewis and others, and this was indeed an early version of the arguments between Schrödinger and Bohr. However, Bohm’s theory does not resolve the measurement problem that grew out of Schrödinger’s cat paper in 1935, and was largely the major obstacle to Einstein’s incompleteness argument regarding the orthodox quantum theory (as will be further examined in Chapter 5) in the sense of his query, “Is the moon there when nobody looks?" Bohm's theory can be considered one out of the many theories that makes predictions to explain precisely under what conditions measurement occurs. In the case of Bohm's theory and the Chapter 3 UMDT, the CHSH sum is predicted to be $2\sqrt{2}$ and entanglement is predicted to persist. However, such quantum states are incompatible with the existence of a particular outcome, which demands a product state representation.