Church of the Higher Hilbert Space

Some Unitarians respond that measurement can always be explained via unitary evolution by the system-plus-device interacting with a larger system. Here two types of unitary theories are examined, 1) unitary theory defined solely by Schrödinger's equation and 2) unitary theory that is necessarily different than that predicted by Schrödinger's equation. Generally speaking, Unitarians that belong to the *Church of the Higher Hilbert Space* invoke a form of Stinespring's dilation theorem.

Von Neumann's original theory [13] was extended to positive operator valued measurement theory by Kraus et al. [49]. Suppose a measurement on a system defined on the Hilbert space $\mathcal{H}_A$ has k possible outcomes**,** and for which there exists a matrix M_k corresponding to the k th outcome that maps any state $\psi \in \mathcal{H}_A$ to $\psi' = M_k\,\psi/||M_k\,\psi||$**.** For each outcome k*,* there exists an associated Hermitian operator of the form $E_k \equiv {M_k}^{\dagger}M_k$ with the properties,

$$\sum E_k = I$$

$$\langle\psi|E_k|\psi\rangle \geq 0 \;\; \forall\; |\psi\rangle \in \mathcal{H}_A,$$

the latter property ensures positivity of the operators. The set $\{E_k\}$ are defined as the elements of the positive operator valued measurement (POVM). When the k th result occurs, the density operator ϱ is transformed according to

$$\hat{\varrho} = \frac{M_k \varrho {M_k}^{\dagger}}{P(k)}$$

with probability $p_k = \mathrm{Tr}\left[M_k \varrho {M_k}^{\dagger}\right] = \mathrm{Tr}[\varrho E_k]$. Note that the set $\{E_k\}$ may not be sufficient to uniquely define the transformation rule of the density matrix as there could be multiple decompositions. Hence it is desirable in the study of the measurement problem to define the set M_k as well. This is further examined as well as various classes of POVMs encountered in Chapter 7. An average density matrix can be found by averaging over all measurement results. This is often referred to as a Kraus decomposition or operator-sum form**:**

$$T(\varrho) = \sum M_k \varrho {M_k}^{\dagger}. \tag{4.10}$$

A completely positive map is a map that remains positive when $T(\varrho)$ is extended to operate on a larger system $\mathcal{H}_A \otimes \mathcal{H}_B$ via $(T \otimes I)\varrho_{A,B}$ where $\varrho_{A,B}$ is a density matrix on $\mathcal{H}_A \otimes \mathcal{H}_B$. It can be shown that T as defined by Equation (4.10) is a completely positive and trace-preserving linear map T. Not all positive maps are completely positive, see Exercise 4.9 in the book or kindle version of theQMP.

Now, suppose one has made a measurement on a system. Then there must exist a completely positive trace-preserving map T that corresponds to the transformation of ϱ

on average. In order to proceed let us consider the following dilation Theorem due to Stinespring [130, p. 22]

> Theorem 4.1: *Given a completely positive, trace-preserving map T between states on a finite-dimensional Hilbert space $\mathcal{H}_A$, there exists an ancilla A defined in an external Hilbert space $\mathcal{H}_B$ and a unitary operation U_S acting on $\mathcal{H}_A \otimes \mathcal{H}_B$ such that $T(\varrho) = Tr_B\big(U_S(\varrho \otimes |0\rangle\langle 0|)U_S{}'\big)$.*

Stinespring's dilation theorem shows that given a measurement that is characterized on average by the operator T, one can always find a unitary operator U and an additional Hilbert space $\mathcal{H}_B$ such that the result of any measurement is the same, on average, as what would be found if the system interacts with an external ancilla via U_S. This implies that T cannot be distinguished on average from that expected under the measurement postulate. Let us consider particle evolution that is evolving via the two cases of 1) Schrödinger unitary evolution and 2) unitary evolution other than Schrödinger's equation.

Schrödinger Unitary

First, the case whereby the unitary is found via Schrödinger's equation is considered. Consider again our UMDT of Chapter 3. It was assumed in Chapter 3 that one can separate the two devices in Figure 3.4 via an arbitrary large distance d, and one could in principle place a shutter in front of the photon emitter to limit the uncertainty of the time of emission to some Δt_e. Assuming $d > 2c(\Delta t_e + \tau_l)$ where τ_l is the latency time of the devices, there can be no direct interaction between the devices such that the state of one device affects the individual statistics of the other device, without violating relativistic constraints of no-signaling. With two devices in the initial state $|\psi_r^0\rangle_1 \otimes |\psi_r^0\rangle_2$ one needs to implement

$$T(\cdot) = \mathrm{Tr}_{A_1}\big(U_S\big(|\psi_{\text{photon,PBS}}\rangle\langle|\psi_{\text{photon,PBS}}| \otimes |\psi_r^0\rangle_1\,{}_1\langle\psi_r^0| \otimes |\psi_r^0\rangle_2\,{}_2\langle\psi_r^0| \otimes |0\rangle\langle 0|\big)U_S{}^{\dagger}\big)$$

where the ancilla is initially in the state $|0\rangle$. However, one cannot implement an arbitrary unitary U that is required under the Stinespring dilation for two separated detectors as direct interaction between the two detectors is not allowed. The best that one can do without violating no-signaling is to implement two Stinespring dilations in which two ancillae are used: A_1 appended to the Hilbert space of Device 1, and A_2 appended to the Hilbert space of Device 2 in the following manner:

$$T_1(\cdot) = \mathrm{Tr}_{A_1}\big(U_{S,1}\big(|\psi_{\text{photon,PBS}}\rangle\langle|\psi_{\text{photon,PBS}}| \otimes |\psi_r^0\rangle_1\,{}_1\langle\psi_r^0| \otimes |0\rangle_{A_1}\,{}_{A_1}\langle 0|\big)U_{S,1}{}^{\dagger}\big)$$

$$T_2(\cdot) = \mathrm{Tr}_{A_2}\big(U_{S,2}\big(|\psi_{\text{photon,PBS}}\rangle\langle|\psi_{\text{photon,PBS}}| \otimes |\psi_r^0\rangle_2\,{}_2\langle\psi_r^0| \otimes |0\rangle_{A_2}\,{}_{A_2}\langle 0|\big)U_{S,2}{}^{\dagger}\big).$$

The following are necessary conditions so that the operations $\{T_1, T_2\}$ are not distinguishable on average from that expected under the measurement postulate: 1) given the photon initial state $|\psi_{\text{photon,PBS}}\rangle = \sqrt{a}|1_V\rangle_B \otimes |0\rangle_C + \sqrt{1-a}\,|0\rangle_B \otimes |1_H\rangle_C$ the state of Device $1'$ must be on average

$$T_1(\cdot) = a\rho_{1,1'}^{(1)} + (1-a)\rho_{2,1'}^{(1)}, \quad (4.11)$$

2) for Device $2'$ on average

$$T_2(\cdot) = (1-a)\rho_{1,2'}^{(1)} + a\rho_{2,2'}^{(1)} \quad (4.12)$$

and 3) the joint state of Device $1'$ and Device $2'$ must either be in the product state $\rho_{1,1'}^{(1)} \otimes \rho_{2,2'}^{(1)}$ or the product state $\rho_{2,1'}^{(1)} \otimes \rho_{1,2'}^{(1)}$ which, on average, is the separable

density matrix given by:

$$a\rho_{1,1'}^{(1)} \otimes \rho_{2,2'}^{(1)} + (1-a)\rho_{2,1'}^{(1)} \otimes \rho_{1,2'}^{(1)}. \quad (4.13)$$

Now, if indeed there is a unitary that is able to interact the detector with an ancilla within the time which can have a causal effect on the measurement outcome, then it must be within a distance of $c(\Delta t_e + \tau_l)$. However, in the construction of the Chapter 3's UMDT, we included in the definition of Device $1'$ and Device $2'$, all particles that are contained within such a causal distance. Hence the ancilla are necessarily part of Device $1'$ and Device $2'$ as we have defined them. Let us consider the main result of Chapter 3. Consider any local Hamiltonian that interacts Mode B and Device 1 and Mode C and Device 2 shown in Figure 3.4. We have proven that entanglement always is predicted under unitary evolution and the CHSH value is $2\sqrt{2}$. Now, the definition of $T_1(\cdot)$ and $T_2(\cdot)$ are unitary, and one can therefore find a corresponding local Hamiltonian that can be used in the Chapter 3 UMDT. The CHSH value will be maintained at $2\sqrt{2}$ with the use of $T_1(\cdot)$ and $T_2(\cdot)$. Hence, it is impossible that the condition that is required in Equation (4.13) is met and the use of the Stinespring dilation fails to resolve the measurement problem.

Note that if one is able to ignore the ancilla, proposals that use such dilations do not appear to be disprovable. For example, consider the proposal by Ozawa in the paper [131] based on operational quantum mechanics. A similar methodology has been proposed by Ozawa based on a dilation to resolve the measurement problem. Ozawa's operational theory is applied to a single detection device and it is shown that, on average, the same density matrix results when using a dilation as would be required under measurement. And for a single measurement device, there is no theoretical or experimental methodology that can discriminate a system for which states occur with a given probability versus a system that always occurs in the same average density matrix. That is, consider an orthogonal decomposition of a mixed state ρ given by

$$\rho = \sum \alpha_i \rho_i$$

where $\alpha_i > 0$ and the ρ_i are orthogonal in the sense that $\mathrm{Tr}({\rho_i}^{\dagger}\rho_j) = 0$. Suppose that a set of particles is prepared in one of two ways by Alice. In the first manner, the particles are always prepared in the state given by $\rho = \sum \alpha_i \rho_i$. In the second manner, the particles are prepared by generating a random variable that corresponds to selection of one of the states ρ_i with probability α_i. In the second manner, each time the particles are prepared, they are in one of the orthogonal states ρ_i. If Alice then gave the particles to Bob without specifying whether or not the particles were prepared via the first method or second method, Bob cannot distinguish the two cases. Furthermore, Alice can give Bob not only a single set of particles, but can provide Bob with many sets of particles so that all cases are either prepared by an average ρ or by selecting a particular ρ_i with probability α_i and Bob still cannot distinguish the two cases.

As long as one is able to ignore the ancilla in the Stinespring dilation, Ozawa's solution cannot be disproven either theoretically or experimentally. When one considers two separated devices as in Figure 3.4, it is possible in principle to detect the entanglement if the interaction is unitary and for which all particles including any ancilla that are within a time-like effect are included in the description, a contradiction with experiment is possible. In such a case, one cannot ignore the ancilla in Stinespring's dilation. This answers a question that was posed in Chapter 2 and leads to the dictum,

> *There is no salvation from the Measurement Problem in the Church of the Higher Hilbert Space.*

Non-Schrödinger Unitary

Having seen that unitary Schrödinger theories that correspond to a particular fixed Hamiltonian cannot possibly be valid explanations of measurement, let us consider unitary theories that are proposed for which arbitrary unitary operations are invoked. Consider the paper by Chirabella et al. [132]. The authors state that they provide a complete derivation of finite dimensional quantum theory from only operational principles.

One might then examine whether or not such operational principles provides a resolution to the measurement problem. The authors develop an operational-probabilistic framework (OPF) based on circuit transformations. The OPF uses a circuit combined with probability theory. The circuits can have classical outcomes, but the theory is used to compute probability distributions for circuit outcomes when the circuits are interconnected.

A circuit or test in the OPF represents a physical device such as a beamsplitter or a photon counter. Each device has an input system and an output with capital letters. The tests have one of several classical outcomes, represented by an index i ∈ X for which each outcome corresponds to a possible event. The set $\{\mathcal{C}_i\}$ of all events from A to B is defined as the set of transformations from A to B, Transf(A,B). Preparation tests are used to define states represented as $|\varrho_i)_B$ as tests with no input (no-input we denote as $\mathcal{I}$). Effects or observations represented as $(a_j|_A$ are defined as tests with no-output. The set of states is denoted St(A)=Transf($\mathcal{I}$,A) and the set of effects Eff(A)=Transf(A, $\mathcal{I}$). The authors show that that composition of a preparation test $|\varrho_i)_A$ with an observation test $(a_j|_A$ gives rise to the joint probability $p_{i,k} = (a_j|\varrho_i)$.

Chirabella et al. in [132] introduce a *purification principle*. A purification of the state $\varrho \in \mathrm{St}_1(A)$ is a pure state $|\Psi_\rho\rangle$ of some composite system AB with the property that ϱ is the marginal of $|\Psi_\rho\rangle$. The purification principle is based on showing that any particular ϱ that exists is equivalent to a unitary operation plus a measurement on the ancilla on a larger Hilbert space with the possible addition of a deterministic effect e_B. The use of a pure state Ψ_ϱ on a larger Hilbert space for which a subsystem has the state ϱ, is termed purification. One can show that purifications exist for any state ϱ and can be accomplished via a unitary theory, for example, via the use of the Stinespring dilation [130, p. 22]. As long as the ancilla or additional environment is included in the Chapter 3 UMDT, the CHSH result is $2\sqrt{2}$ under unitary evolution. However, if the system is in a definite state, then the CHSH result (including the additional environment or ancilla required to accomplish the purification) is $\sqrt{2}$.

Chirabella et al state in [132]:

> *... purification implies the possibility of simulating any irreversible process through a reversible interaction of the system with an environment that is finally discarded.*

Indeed, one is capable of simulating an irreversible process, but only when the environment is discarded. If the environment were included, there would be a reversible process and the entanglement would be maintained in the Chapter 3 UMDT test. The point is that when the additional environment is included, the CHSH result must be $\sqrt{2}$ when a measurement occurs, but will always be $2\sqrt{2}$ under unitary evolution. One cannot simply ignore the additional particles required to accomplish the purification in the UMDT test of Chapter 3, because they are well defined and limited in number by the condition $d > c(\Delta t_e + \tau_l)$. In terms of the measurement process, the authors state:

> *The purification principle expresses a law of conservation of information, stating that at least in principle, irreversibility can always be reduced to the lack of control over an environment. More precisely, the purification principle is equivalent to the statement that every irreversible process can be simulated in an essentially unique way by a reversible interaction of the system with an environment, which is initially in a pure state. This statement can also be extended to include the case of measurement processes…*

Solutions such as this are dispersing the entanglement into the environment to try and force a unitary solution. But even if the entanglement disperses into the environment—it hasn't gone away and no measurement can be said to have occurred unitarily. The solution proposed in [132] violates our dictum:

To the extent there is entanglement, there is no measurement.

Another result of OPF [132] is for any two states ϕ_0 and ψ_0 there exists a unitary transformation $U \in \mathrm{Trans}(\mathrm{A},\mathrm{B})$ such that $U\phi_0 = \psi_0$. This is correct; there always exists a unitary transformation that maps one state to another. Consider again the Chapter 3 UMDT and for simplicity that the two devices are 100% absorbing and initialized to the pure initial states $|\psi_r^0\rangle_1$ and $|\psi_r^0\rangle_2$. Device 1 (Device 2) evolves to $|\psi^1\rangle_1$ ($|\psi^1\rangle_2$) when a photon is detected and to $|\psi^0\rangle_1$ ($|\psi^0\rangle_2$) when no photon is detected.

Suppose that $U_1|\psi_r^0\rangle_1$=$|\psi^1\rangle_1$ and $U_2|\psi_r^0\rangle_1 = |\psi^0\rangle_1$. Then there are reversible transformations that will take the initial state of Device 1 (and similarly for Device 2) to either the state of Device 1 representing a detection or the state of the device representing no-detection. If one knows which device detects the photon, then one could choose the correct unitary that corresponds to the action that occurred during measurement.

In [132] a unitary description of measurement is proposed. While the authors are correct in that such a choice of unitary operations can be done *a posteriori*—such a specification requires knowledge of which device detected the photon. Such *a posteriori* information of which device detected the photon, demands information regarding the *a priori* statistics of the degree of superposition of the photon, which is parameterized by the variable *a*. A methodology of applying either U_1 or U_2 depending on *a posteriori* information is not a valid solution to the problem, because prior to the measurement occurring, there is only a fixed Hamiltonian with a fixed unitary evolution. Schrödinger unitary evolution is determined *a priori* to measurement. Initially, when the photon is launched through the beam splitter, neither the Hamiltonian of Device *1* nor Device *2* can be a function of *a* without violating causality. The issue becomes whether or not such an *a priori* Hamiltonian can adapt itself to become a function of the degree of superposition *a* by interacting with the photon, that has a state that is a function of *a*. In order for this to occur, it must be possible locally at Device *1* and Device *2* for the devices themselves to be able to determine the superposition *a*, and then modify themselves in a manner to account for the evolution. However, such a device would need to be able to extract the degree of superposition *a*, but this would violate the no-cloning theorem [54] for which it is impossible to build any device that can reliably determine *a*.

The operations that would be needed by OPF to specify these unitary operators would, if Schrödinger's equation were obeyed, require the devices in the measurement problem setup to have either a time-varying Hamiltonian that depends on the degree of superposition *a*, or an environment that is a function of *a*, i.e., via a purification or

Stinespring dilation. Again, this is impossible without violating causality. It does appear to be possible to develop a set of reversible unitary operations *a posteriori* after knowing the measurement results, which is consistent with a statistical or a non-unitary evolution. Papers occasionally appear in the literature that do exactly this as an approach to resolving the measurement problem. A recent example for which the authors have defined a unitary operator as a function of the measured state can be seen in Equation 11 of [133] by Cruikshank and Jacobs. These same authors' purport, "If a physical measurement-based process is designed to produce a single outcome, then even the measurement at the end is unnecessary." Such a solution may be satisfactory to resolve the philosopher's version of the measurement problem, but it does not resolve our version: such unitary operators cannot follow from Schrödinger's evolution that requires a Hamiltonian to be specified prior to the act of measurement.

One can't simply throw around unitary operators here and there and claim that Nature is describable by a reversible *a priori* theory. Simply because there exists a set of unitary operators that can describe a given evolution, given the *a posteriori* final state, does not imply that the same quantum mechanical evolution follows from the unitary evolution *a priori* that is predicted by Schrödinger's equation with a time invariant and causal Hamiltonian for which the *a posteriori* final state is unknown.

Unitary theory combined with Kraus operators or measurement can be described *a posteriori* by reversible unitary quantum operations. This may be quite correct, but such unitary operations provide no *a priori* predictive power, which is a basic requirement of any scientific theory and are not derivable from Schrödinger's equation within a causal Hamiltonian formulation.