Continuous Spontaneous Localization (CSL)

The discrete time localization theory proposed in the original paper [224] was later extended in the paper [226], in which the time of the jumps is decreased to zero via the use of a stochastic differential equation. It has been proven that under certain conditions the stochastic Brownian motion is almost surely continuous. This means that, unlike the original GRW proposal, CSL appears to move in a continuous manner. However, such processes are also generally almost surely non-differentiable [245, p. 3]. In CSL a locally averaged density operator is defined as

$$N(\boldsymbol{x}) = \sum_{s} g(\boldsymbol{y} - \boldsymbol{x}) a^{\dagger}(\boldsymbol{y}, s) a(\boldsymbol{y}, s) d\boldsymbol{y}$$

where $a^{\dagger}(\boldsymbol{y}, s)$ is a particle creation operator at point $\boldsymbol{y}$ and spin s and $a(\boldsymbol{y}, s)$ the respective destruction operator, $g(\boldsymbol{x})$ is a spherically symmetric positive real function that integrates to unity and is assumed to be a zero-mean Gaussian distribution with standard deviation σ. An advantage of [226] is that reduction preserves the symmetry properties of identical particles, whereas the original proposal did not maintain symmetry. Equation (4.28) can be extended to a summation over operators A_i,

$$d|\psi_t\rangle = -\frac{i}{\hbar} H|\psi_t\rangle dt - \frac{1}{2}\sum_{i} A_i{}^{\dagger} A_i |\psi_t\rangle dt + \sum_{i} A_i\, |\psi_t\rangle dB_t.$$

Associating the operators A_i with the operator $N(\boldsymbol{x})$ and the index $\boldsymbol{i}$ with the space point $\boldsymbol{x}$ and converting the summations to integrals in the infinite dimensional case, it is found [244] that the equivalent Itô equation is given by

$$d|\psi_t\rangle = -\frac{i}{\hbar} H|\psi_t\rangle dt + \int N(\boldsymbol{x})|\psi_t\rangle\, dB(\boldsymbol{x}) d\boldsymbol{x}\, dt - \frac{1}{2}\int N^2(\boldsymbol{x})\, |\psi_t\rangle d\boldsymbol{x} dt$$

where $\overline{dB(\boldsymbol{x})} = 0, \overline{dB(\boldsymbol{x}) dB(\boldsymbol{y})} = \delta(\boldsymbol{x} - \boldsymbol{y}) dt.$

Let us suppose that the general form of the solution to the measurement problem to be of the form described by a Schrödinger equation plus an additional term that allows variation from the purely Schrödinger evolution via wave function coupling to a noise term. In such a case, it has been proven in [260, pp. 165-175] that, up to the choice of operator to which the noise is coupled, the CSL theory is a unique form that yields the Born rule as a deduced consequence.

Criticisms of CSL

CSL theory has remedied some problems that the original GRW suffered. CSL preserves the symmetry properties of the particles. However, CSL suffers from many similar problems as the original GRW. The original GRW was parameterized by the two parameters λ_h and σ_h. In CSL there is no reason to consider λ_h as the localization process is continuous, rather only the parameter of the standard deviation of the Gaussian $g(\boldsymbol{x})$ is relevant.

Additionally, GRW is in principle reversible and is also a time-symmetric collapse theory [261]. On the other hand, as we have already discussed, measurement is sufficient but not necessary for loss of coherence within the von Neumann theory. That is, the absorption of a photon by one of many possible detectors is, within the von Neumann theory, generally an irreversible action that destroys the phase coherence of the photon and is not generally time-reversible. Such time-reversible theories are not consistent with this fundamental theoretical tenet. Within von Neumann theory, it is theoretically impossible to reverse a photon that has been positively measured by a specific detector. There have been experiments that have reversed unitary evolution, but this is hardly different than throwing a ball up in a gravitational field and having it come down. Nobody has yet reversed a photon that has been positively measured in the measurement process, and to expect otherwise is due the improper application of inductive logic to a problem that demands a deductive approach, which will be further discussed in Chapter 6.

The remaining criticisms of GRW appear to go over into CSL such as lack of energy and momentum conservation as well as the issues with the tail of the Gaussian.