External Orthogonalization

A unitary methodology of external orthogonalization that is within the Hamiltonian formulation is presented, which will be useful in analyzing numerous approaches that have been proposed to resolve the measurement problem. Consider the Hamiltonian consisting of terms of the system to be measured H_S, a set of external particles with Hamiltonian H_D with an orthonormal set of eigenstates $|\Psi_r\rangle$ and an interaction Hamiltonian H_{int} between the system and the set of external particles. The external particles could, for example, compose a device or be part of the local environment surrounding the system and are assumed initialized to state $|\Psi_0\rangle$. The interaction Hamiltonian will be constructed in a manner that if a system is in the r th eigenstate of H_S denoted $|\phi_r\rangle$ then the external set of particles will make a unitary transition from $|\Psi_0\rangle$ toward populating the r th eigenstate of the external particle set $|\Psi_r\rangle$.

This method will be referred to as *external orthogonalization* because each system eigenstate $|\phi_r\rangle$ is paired with a corresponding external state $|\Psi_r\rangle$, with the property that the $|\Psi_r\rangle$ form an orthonormal set. A Hamiltonian that accomplishes this is given by

$$H = H_S \otimes I + I \otimes H_D + H_{int} \tag{4.1}$$

$$H_S = \sum_r E_S |\phi_r\rangle\langle\phi_r|$$

$$H_D = \sum_r E_P |\Psi_r\rangle\langle\Psi_r|$$

$$H_{int} = \sum_r (|\phi_r\rangle \otimes |\Psi_0\rangle)(\langle\phi_r| \otimes \langle\Psi_r|) + (|\phi_r\rangle \otimes |\Psi_r\rangle)(\langle\phi_r| \otimes \langle\Psi_0|).$$

In order to illustrate this concept, it is assumed that the energies are the same E_S for all $|\phi_r\rangle$ and as well the pointer states $|\Psi_r\rangle$ all have energy E_P. If the system is allowed to evolve for a time $t = \pi/2$, the unitary evolution operator

$$U\left(\frac{\pi}{2}\right) = e^{-\frac{iH\pi}{2}}$$

performs the desired operation. If the system is initially in the superposition given by

$$|\psi_S\rangle = \sum_r c_r |\phi_r\rangle$$

then $U(\pi/2)(|\psi_S\rangle \otimes |\Psi_0\rangle)$ is given by

$$\sum_r c_r e^{i\theta_r} |\phi_r\rangle \otimes |\Psi_r\rangle. \tag{4.2}$$

This method will be referred to as external orthogonalization because each system eigenstate $|\phi_r\rangle$ is paired with a corresponding external state $|\Psi_r\rangle$, with the property that the $|\Psi_r\rangle$ are an orthogonal set.

If the energies E_S and E_P are the same for all $|\phi_r\rangle$ and $|\Psi_r\rangle$, the phase θ_r is independent of r. The term $e^{i\theta_r}$ factors out and, since the overall phase of the state has no observable consequences, the term $e^{i\theta_r}$ can be dropped from the expression, resulting in:

$$\sum_r c_r|\phi_r\rangle \otimes |\Psi_r\rangle.$$

One can trace the external particles from the overall state and compute the partial density matrix of the system. This is seen to be the diagonal matrix:

$$\begin{pmatrix} |c_1|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |c_N|^2 \end{pmatrix}. \qquad (4.3)$$