Mass Threshold Theory

Consider a pure state of a matter wave function that enters an interferometer. For example, consider a molecule that interacts with a beam splitter mechanism that causes the molecule to separate into two paths. If the general state of the molecule was not preserved, it is possible that one could find the molecule dissociated into various atoms in the two arms when measurement occurs that localizes the molecule to one or the other arm. However, this is not the case—such molecules are found to be entirely in one or the other arm when measured.

Interference experiments are being performed with larger and larger systems. In 1999 an experiment showed that the fullerene molecule C_{60} exhibited interference [23]. In 2011 organic molecules of sizes up to 6 nm composed of 430 atoms were shown to exhibit interference [262]. Also demonstrated was the resilience of such molecules to resist decoherence. In 2013, [26] demonstrated high-contrast quantum fringe patterns with molecules composed of 810 atoms consisting of approximately 5000 protons, 5000 neutrons, and 5000 electrons, with a large number of possible internal energy levels. In 2010 [263] [264] and more recently 2016 [265], the possibility of creating superpositions of living organisms has been suggested using quantum optomechanics. In [265] it was suggested that the range of viruses of 10^{-17}grams to bacterium on the order of 10^{-12}grams could be experimentally probed. In [266] a measure has been proposed for quantifying the macroscopic size of a quantum superposition. The authors also review technologies that have enabled larger and larger superpositions to be experimentally probed resulting in a plethora of advances on the macroscopic scale between 1980 and 2013.

With such advancements, one might expect that there is no inherent limitation on the size of a spatial superposition. Although no inherent limitation has yet been seen experimentally, there is already an answer to this question.

Consider a mesoscopic object of mass m that can take two paths through a double-slit of size d as shown in Figure 4.2. Such a device was considered by Feynman in 1957 [267] when the mass m is sufficiently large that the gravitational field can be used to discern which-way information and by Bohr [268] if an object with a sufficiently large charge can be used to discern which-way information via the Coulomb field.

A device of mass M is located at a distance R from Slit 2 and at a distance $R+d$ from Slit 1. The force on the device if the object were to pass through Slit 1 is given by $F_1 = GmM/(R+d)^2$ whereas the force on the device if the object were to pass through Slit 2 is given by $F_2 = GmM/R^2$. Suppose that no device is able to make a measurement such that the force can be inferred to an accuracy smaller than $F_2 - F_1$. In such a case it is possible for the particle to exhibit interference. We will denote this case as *below threshold*, and the interference pattern for the below threshold case is shown in Figure 4.2. Suppose that a device exists that is able to make a measurement such that the force can be inferred to an accuracy smaller than $F_2 - F_1$. In such a case, in-principle which-way information could be obtained and first-order interference is eliminated. Note that it is not required that one actually obtains distinguishing which-way information via an actual measurement—it is only required that one could obtain such which-way information. Such cases will be referred to as *above threshold*. This shows that once the mass of the object *m* (or possibly mass in-combination with other parameters) is such that the gravitational field of the object can be determined in

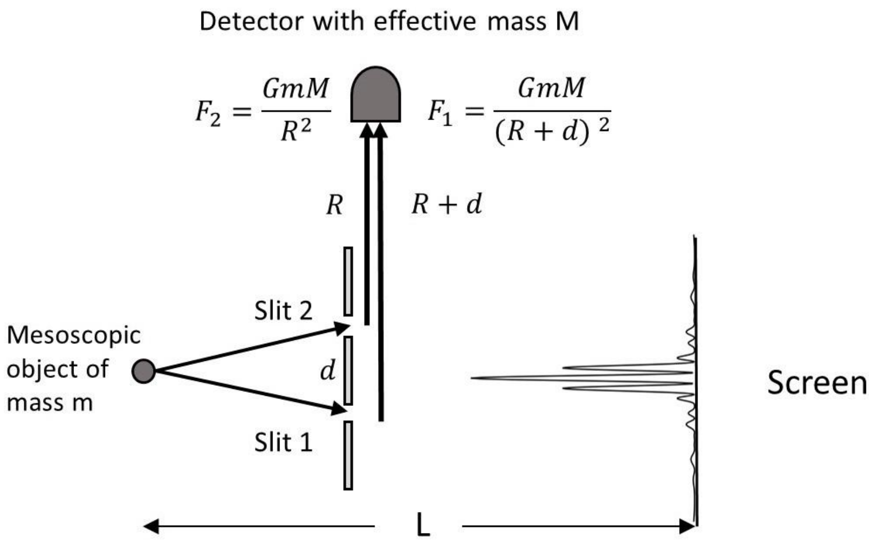

Figure 4.2: Double-slit experiment with gravitational detectors, below the detection threshold, double-slit interference pattern is retained.

principle by a remote detector that is capable of resolving whether or not the object passed through Slit 1 versus Slit 2, then interference through the interferometer will be lost.

An analysis has been given by Feynman [267] for the case where the particle is detected based on its mass. If a measurement occurs in a cubic volume l^3 in a time given by l/c, the dimensionless gravitational potential (which is normalized by dividing the non-dimensionless gravitational potential by c^2) will remain uncertain to [269, p. 1192] [270]

$$\begin{aligned}\Delta g &= \sqrt{\frac{\hbar G}{c^3 l^2}} \\ &= \frac{l_p}{l}.\end{aligned} \tag{4.36}$$

where $l_p = \sqrt{\hbar G/c^3}$ is the Planck length. In order to detect the mass of an object, the gravitational potential $g(x)$ must be greater than Δg within the detection volume l^3. The dimensionless gravitational potential produced by an object of mass m at a distance x from the object is given by

$$g(x) = \frac{Gm}{xc^2}. \tag{4.37}$$

Assuming the object is placed outside the detection volume, if the minimum gravitational potential due to the object that is within the detection volume is greater than uncertainty in the gravitational field Δg, then the object should be detectable. If the object is placed on the boundary of the detection volume, then the minimum gravitational potential due to the object would be (for a two-dimensional detector) at a distance l. In such a case, the gravitational potential is always greater than $g(l)$ in the detection region. As an uncertainty in the gravitational potential is equivalent to an uncertainty in the mass, one finds in detectable regions

$$\Delta m \geq m_p \tag{4.38}$$

where $m_p = \sqrt{\hbar c/G}$ is the Planck mass. Hence it appears from this argument that the detectable mass in time l/c should be greater than the Planck mass m_p.

Note that as one places the object farther from the detection volume, the minimum gravitational potential will decrease, hence the maximum (with regard to distance from the detection volume) over the minimum (with regard to within the detection volume) occurs when the object is on the boundary. The Planck mass m_p is approximately 0.02 mg, which is quite large in comparison with the mass of most typical atomic particles (e.g.10^{19} times the proton mass). It would appear that if a mass has an uncertainty that is less than the Planck mass, that such a mass would have a gravitational potential that necessarily falls below Δg somewhere within the detection volume and may not be detectable. It appears a reasonable hypothesis that a single Planck mass is on the threshold of being externally reliably detectable by its gravitational field. Hence it is no surprise that atomic particles are observed to often exist in coherent first-order superpositions of position. Also note that as the reduced Compton wavelength of a mass m is $\lambda_c^{(r)} = \hbar/(mc)$, then $m = m_p$, it is seen that $\lambda_c^{(r)} = l_p$ or approximately 1.6e-35 m which is quite small compared with typical atomic particles. Hence a Planck mass has a rather large mass in comparison with the typical mass of atomic particles and if such a mass were to be confined to a Planck length, would have an inordinately high density.

Although this experiment has not yet been accomplished, related experiments to test for collapse once the mass m is sufficiently large have been proposed by [271] [272] [273] as well as others to determine if quantum superposition features of the gravitational field can be revealed. Although experimental methods are improving, the testing of gravitational superpositions has not yet been achieved due to the relatively weak coupling in comparison to electromagnetic coupling. Many have been touting the benefits of quantum optomechanics for the purpose of probing quantum macroscopic superpositions [274] [275].

A black hole is predicted to form if a mass to be confined to a sphere of radius smaller than the Schwarzschild radius given by $r_s = 2GM/c^2$. The Schwarzschild radius of a Planck mass is $r_s = 2l_p$. A limitation of a particle mass m is (approximately) that its size is greater than $\lambda_c^{(r)}$. Hence it appears that a Planck mass that is confined to its reduced Compton radius is not only at the limit of external gravitational detectability but also at or nearby the threshold of black hole formation.

A bound similar to Equation (4.38) has also been derived based on a derivation of the uncertainty of the measurement of an electric field that was developed by Bohr and Rosenfeld [276]. Bohr and Rosenfeld utilized a test charge within the detection volume that would be subject to a change in momentum depending on the electric field. Their relationship was extended to the gravitational field by Peres and Rosen [277] which yields the result for the gravitational potential

$$\Delta g \geq \left(\frac{l_p}{l}\right)^2.$$

Furthermore, [278] propose a lower bound given by,

$$\Delta g \geq \left(\frac{l_p}{l}\right)^{\frac{2}{3}}.$$

Note that as $\Delta g \leq 1$, the largest lower bound is given by [278] followed by the bound proposed by Feynman and lastly the bound of Peres and Rosen based on the work of Bohr and Rosenfeld. It is shown in [278] that using their improved bound, the minimal mass for which the measurement time is less than the expected decoherence time is also the Planck mass. Hence by many accounts, the Planck mass is a reasonable hypothesis for establishing the classical-quantum gravitational boundary.

Baym and Ozawa [268] have analyzed a modern gravity detector that is sensitive to changes in the gravitational field. The detector consists of two mirrors that form a laser interferometer that are separated by a distance $S + \eta(t)$. When the gravitational potential $\phi(x,t)$ is zero $\eta(t) = 0$, but when $\phi(x,t)$ changes, there is a force exerted that changes the distance between the mirrors. It is assumed that $\eta(0) = d\eta/dt|_{t=0} = 0$ and at the time of the measurement t_f the potential has changed to a value $\phi(x,t_f)$ and is constant during the measurement. Under these conditions, the equation for $\eta(t)$ is given by [268] :

$$\eta(t) = -\frac{d^2}{dt^2}\phi(x_0,t)S\frac{1-\cos wt}{w^2} \tag{4.39}$$

where x_0 is the midpoint between the mirrors. The accuracy of the device that is required in order to discriminate whether the object passed through Slit 2 versus Slit 1 is given by

$$\Delta\frac{d^2}{dt^2}\phi(x_0,t) = 2Gm\left(\frac{1}{R^3} - \frac{1}{(R+d)^3}\right)$$

$$\sim \frac{Gmd}{R^4}.$$

Using the relation $(1 - \cos wt) \leq (wt)^2/2$, it is seen that

$$\Delta\eta(t) \leq \frac{GmdST^2}{R^4}.$$

It is assumed $R > cT$ where T is the time-of-flight of the object so that the detector is sufficiently distant from the apparatus that there is no back-reaction from the measurement and hence,

$$\frac{GmdST^2}{R^4} < \frac{GmdS}{R^2c^2}.$$

Assuming $S \ll R$

$$\Delta\eta(t) < \frac{Gmd}{Rc^2}.$$

Squaring both sides and multiplying the top and bottom of the left-hand-side by $\hbar$ gives

$$\frac{\hbar\Delta\eta^2(t)}{\hbar G^2} < \frac{m^2}{c^4}\left(\frac{d}{R}\right)^2$$

and substituting the Planck length $l_P = \sqrt{G\hbar/c^3}$ results in

$$\frac{\hbar c\Delta\eta^2(t)}{{l_P}^2 G} < m^2\left(\frac{d}{R}\right)^2.$$

There are several compelling reasons presented in [279] for believing that $\Delta\eta(t)$ must exceed the Planck length. Assuming this is the case,

$$\frac{\hbar c}{G} < m^2\left(\frac{d}{R}\right)^2.$$

Substituting the Planck mass $m_p = \sqrt{\hbar c/G}$

$$m > m_p\left(\frac{R}{d}\right). \qquad (4.40)$$

We define $m_{th} \equiv m_p R/d$. The Planck mass $m_p = \sqrt{\hbar c/G}$ is approximately 2 x 10^{-5}g and for R/d on the order $1/m_p$ gives a mass on the order of 1 gram. The fringe separation seen on a screen that is a distance L from the particle is given by [268]

$$d_f \approx \frac{L}{d}\frac{\hbar}{mv}.$$

Assuming the velocity $v \approx L/T$ and $T \approx R/c$ gives

$$d_f \approx \frac{\hbar R}{cdm}.$$

Substituting into Equation (4.40) gives

$$d_f \approx \frac{\hbar R}{cdm} < \frac{\hbar}{cm_p}.$$

Noting that for $\hbar/(cm_p) = l_P$ it is found that an object with mass $m > m_{th}$ that satisfies Equation (4.40) also has an associated fringe separation that satisfies (subject to the approximations made in [268])

$$d_f < l_P.$$

This indicates that whereupon the mass is great enough that it can be detected by an external gravimeter, the fringe separation that could possibly be used to detect first order coherence of the object becomes less than the Planck length and is undetectable.

Gravitationally Induced Collapse

Penrose [280] has put forward a mass threshold theory of gravitationally induced collapse termed objective reduction. Penrose considers an object of mass m that unitarily evolves to a superposition of two locations f_1 and f_2 that are separated by a given distance $|f_2 - f_1| = d_p$. Although such a superposition is stationary in standard quantum mechanics, Penrose proposes that such a superposition is ultimately not stationary due to the existence of two different gravitational fields that define two different space-time structures. Penrose proposes that the existence of two different space-time structures is untenable and hence such a superposition will decay to one of the two locations f_1 or f_2. Penrose proposes that the characteristic decay time is given by

$$\tau \cong \frac{\hbar}{E_G} \tag{4.41}$$

where E_G is the gravitational self-energy of the difference of the two locations, which is equivalent to the energy required to separate two instances of the object initially at a common point and then moved to the two locations f_1 and f_2. The same time scale has been proposed by Diosi [281] and it is claimed by Diosi [282] that his and Penrose's scheme yield similar results, although Diosi's theory is based on a dynamical master equation whereas Penrose assumes a single characteristic time for collapse. Now, if such a collapse mechanism exists and is verified by loss of first-order coherence to occur on a time scale on the order of $\hbar/E_G$, *and* this is different than predicted by the proposed non-local unitary quantum theory, then the necessity of adding an additional collapse mechanism would be warranted. On the other hand, if the unitarily predicted quantum state itself, when including a gravitational field that becomes externally detectable, leads to a similar finding of loss of first-order coherence, then more work would be needed to ascertain whether or not a collapse mechanism is necessary to explain the phenomenon observed in an experiment. That is, a unitary explanation might not be ruled out, depending on the experiment and the parameters involved.

A review of other theories of collapse, including gravitational collapse is given in Bassi et al. [283]. Various models of gravitationally induced decoherence are contrasted in Bera et al. [284]. Diosi [285] expects that the natural width a_0 of the wave packet of any macroscopic object is

$$a_0 \approx \frac{\hbar^2}{Gm^3}.$$

This estimate also coincides with work of Károlyházy in [286].

Note that as has been previously discussed, such collapse has also been proposed to be associated with the formation of consciousness in the Penrose-Hameroff orchestrated reduction model. In this model, the objective reduction in gravitational collapse is orchestrated or mediated within microtubules which also results in consciousness.

Quantum Mechanics with Fields

There must be inherent limitations of a spatial superposition, with all other parameters held constant. This is because there exist devices such as gravimeters that can measure differences in gravitation from mass variation. There is no doubt that a particle will eventually lose first-order coherence once $m > m_{th}$.

Typically, when one computes a first-order interference pattern in quantum mechanics, one evolves the local particle state that moves through the interferometer and ignores the gravitational field. Baym and Ozawa [268] re-examined Bohr's analysis of two-slit interference with charged particles and the implications for quantization of the electromagnetic field as well as the gravitational counterpart to such an experiment using massive particles. The Baym-Ozawa work does not explain how to compute the first-order interference pattern from standard quantum mechanics—only that the interference fringes are not in principle measurable when $m > m_{th}$. However, when the gravitational field can be detected and the information can discern which-path information, this indicates that standard local non-relativistic quantum mechanics requires modification. Feynman stated at a 1957 conference at

Chapel Hill [267]

> *One can conclude that either gravity must be quantized because a logical difficulty would arise if one did the experiment with a mass of order $10e-4$ grams, or else that quantum mechanics fails with masses as big as 10e−5 grams.*

We show here why this is and examine the possibilities in modifying the standard local non-relativistic Schrödinger unitary evolution to a generalized non-local non-relativistic Schrödinger unitary evolution. Let us consider a point (x_s, t) that is located a vertical distance d_s from the peak of the pattern on the screen in Figure 4.2. The state at x_s after the two slits can be examined by considering the phase coherence of the vector $|\psi\rangle = [\psi_1(x_s, t)\ \psi_2(x_s, t)]^T$, where ψ_1 represents the wave function that travels only through Slit 1 and ψ_2 through Slit 2, T denotes the transpose operation. In the von Neumann theory, these vectors actually exist and change in time. However, from the Baym-Ozawa result they are not observable when $m > m_{th}$ due to limitations of measurement devices which are imposed largely by uncertainty relationships. Now $|\psi\rangle = [\psi_1(x_s, t)\ \psi_2(x_s, t)]^T$ will generally have some phase relationship, for example

$$|\psi\rangle = [\sqrt{a}\, e^{-iwt}\ \sqrt{1-a} e^{-iwt+\theta}]^T.$$

Computing the corresponding density matrix gives

$$\rho(x_s) = \begin{pmatrix} a & a(1-a)e^{-\theta} \\ a(1-a)e^{\theta} & 1-a \end{pmatrix}.$$

When an object's mass exceeds m_{th}, the Baym-Ozawa result shows that the fringe separation is less than the Planck length, and there can be entire oscillations of fringes within a single Planck length. Since the fringes are oscillating, the density matrix is a function of x, due to alternating constructive interference and destructive interference. In such a case, if a physical detector integrates over a size greater than l_P, then two-slit interference would be expected to be lost. What we are left with as seen by a detector of size greater than l_P is essentially

$$\rho(x) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}. \qquad (4.42)$$

Remarkably a detector in this case can only see an effective mixed state of the system. Yet Schrödinger's equation tells us that a pure state should exist for an object in isolation, unless the object cannot be considered to be in isolation. And this provides an explanation as to precisely what has happened. When an object's mass exceeds m_{th}, the object cannot be treated as if it is in isolation. The gravitational field of the object now allows in-principle discrimination of the which-way path information that the object took.

The state of the problem (without assuming collapse) is suddenly enlarged when the mass exceeds m_{th}. Whereas the object could previously be considered to be an isolated system described by a pure state that exhibits first-order interference when $m < m_{th}$, the object can no longer be considered an isolated system when $m > m_{th}$; additional degrees of freedom now need to be appended to the system for its full description. In this case, the detectable gravitational field needs to be considered.

Let us denote the state of the gravitational field at the detector in Figure 4.2 as $|\phi_1\rangle$ if the object travels through Slit 1 and as $|\phi_2\rangle$ if the object travels through Slit 2. Then the overall state of the particle plus its gravitational field is $|\psi\rangle = (|1\rangle|\phi_1\rangle + |2\rangle|\phi_2\rangle)/\sqrt{2}$ and one finds that the unitary prediction of the state of the particle is entangled with its gravitational field. In such a case when one traces the system from the entangled state it is found that $\rho(x)$ is mixed in agreement with Equation (4.42).

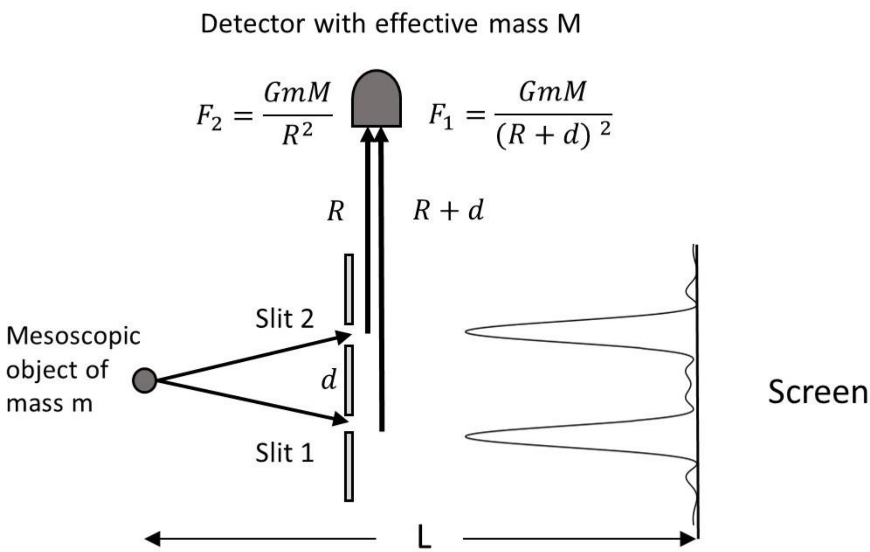

Figure 4.3: Double-slit experiment with gravitational detectors, above detection threshold allowing which-way information, double-slit interference pattern eliminated.

This shows that an object with mass in quantum mechanics cannot simply be considered to be in the Hilbert space spanned by the states $|1\rangle$ and $|2\rangle$ but rather the particle's gravitational potential must be included so that the state of the particle cannot be considered as a local state for the purpose of making calculations. The Hilbert space must be enlarged to include both state of the local particle and its gravitational potentials to the extent that they are externally detectable,

$$|\psi\rangle = (|1\rangle|\phi_1\rangle + |2\rangle|\phi_2\rangle)/\sqrt{2}.$$

Note that if the gravitational potentials are not detectable, then the extent that they are detectable is the same for both $|1\rangle$ and $|2\rangle$ and the state $|\phi_1\rangle = |\phi_2\rangle$. The state $|\phi_1\rangle$ is not the gravitation potential but represents a state of the gravitational potential to be externally detectable and differentiated from $|2\rangle$. When $m > m_{th}$, the states must be included and so long as the uncertainty in the gravitational field produced when the particle is locally in state $|1\rangle$ versus state $|2\rangle$ is less than the measurable uncertainty equivalent to a single Planck mass, then the states are orthogonal, i.e., $|\phi_1\rangle \perp |\phi_2\rangle$. In such a case, any measurements of the local particle states that might reveal first-order local coherence would be impossible as the local particle state is now a mixed state and the interference pattern of a double slit experiment shown in Figure 4.3 results.

In terms of the precise unitary physics one might expect that each term $|1\rangle|\phi_1\rangle$ and $|2\rangle|\phi_2\rangle$ evolve according to Schrödinger's equation. The fields $|\phi_1\rangle$ and $|\phi_2\rangle$ are quantum objects in the sense that they could appear or disappear depending on what measurements are made on the particles $|1\rangle$ and $|2\rangle$. However, so far, there has not been a need to quantize the fields, that is, they are continuous and individual gravitons are not needed to establish this argument yet. Interestingly, when Feynman in 1957 stated the requirement that the gravitational field would need to be quantized, Rosenfeld replied [267],

> *I do not see that you can conclude from your argument that you must quantize the gravitational field. Because in this example at any rate, the quantum distinction here has been produced by other forces than gravitational forces*.
>
>

The interaction of quantized and non-quantized systems has been further considered by Peres [287]. Others such as in [288] [289] have considered methods of providing a quantum state representation that includes both matter and gravitational states. Often such quantum models that incorporate gravity are being developed to understand processes such as black-hole formation and the formation of event horizons. Further work along these directions is needed to determine unitary predicted evolution which is required to discern unitary predictions from measurement collapse models.
As shown in Chapter 3, under unitary evolution first order coherence is simply replaced by higher order coherence or entanglement. An issue addressed in Chapter 3 is how to experimentally discriminate entanglement from a product non-entangled state that is predicted under measurement. The problem at hand is somewhat similar in that first-order coherence of the object has now been unitarily replaced by object-gravitational field entanglement coherence.

A bona fide measurement requires that the state of the particle and field be either $|1\rangle|\phi_1\rangle$ or $|2\rangle|\phi_2\rangle$, i.e., particle-field are in a product state. That is, the particle travels through only one slit and the field exists as if the particle took the respective path. On the other hand, the unitarily predicted solution for this problem is non-local, and similar to the unitary analysis of detection in Chapter 3 that predicts an entangled state of particle-path and gravitational field. Experiments that investigate the loss of first-order coherence of the particle and not the loss of entanglement required in collapse may not be sufficient to resolve these issues. It is possible that experiments show the eventual loss of first-order coherence, which is predicted under *both* measurement and our proposed non-local extension of unitary quantum mechanics. If one can experimentally quantify entanglement due to gravitational superpositions such as proposed in [273] [290], then there still remains the possibility that measurement cannot be distinguished from our proposed non-local extension of unitary quantum mechanics if the loss of entanglement due to a physical collapse is precisely concurrent with the theoretical external detectability of the difference in the gravitational fields of the two objects. On the other hand, if the loss of entanglement due to a physical collapse occurs below the threshold for which exists theoretical external detectability, then such an experimental endeavor could prove to be successful.

Experiments on superpositions in gravity are nonetheless important, but there is little doubt that first order coherence must eventually be lost as m increases. Whether entanglement is also lost for $m > m_{th}$ is unknown. That is, whether or not the final state is $|\psi\rangle = (|1\rangle|\phi_1\rangle + |2\rangle|\phi_2\rangle)/\sqrt{2}$ versus the state being identically either $|1\rangle|\phi_1\rangle$ or $|2\rangle|\phi_2\rangle$ in the case of measurement, does not appear to be a simple matter to discern.

If it is not plausible to directly discern entanglement from a product state when gravitational fields are involved, a manner in which at least indirect evidence could be obtained that could provide support to a detailed theory is that the theory provides a specific collapse model that is different than predicted under unitary evolution. For example, if the lifetime of a superposition that is proposed under Penrose's collapse proposal is verified and is different than that under unitary evolution, or a theory predicts loss of first-order coherence before unitary evolution makes such a prediction, then such results would have to be taken into account as providing indirect evidence in support of that theory.

Related issues concerning entanglement in terms of gravity as regards to such measurement theories have been examined in [291] for which they claim that gravitational decoherence models given by [292] cannot be correct. If it were to be proven to be impossible to discern entanglement from product state measurement, and no other parameters of a proposed theory are distinguishable from unitary theory, this would render such a proposed theory a Category 2 theory, which will be discussed later.