Wave Function Reduction

Measurements are generally of one of two types. In one type, within the process of measurement, the particle that is being measured is annihilated entirely and completely absorbed within the measurement device. The second type of measurement leaves the system particle intact but can serve to reduce or affect the wave packet. Quantum non-demolition measurement is a form of the second type of measurement in which it is attempted to minimally reduce or affect the wave-packet in the process of obtaining information regarding the particle. In this section, we will restrict attention to the latter process whereby the system particle remains intact in the process of interaction with the measurement device.

When an individual electron is emitted, it exists with energies that are quantized and propagates as a wave and exhibits path interference. In [7], Heisenberg examined the problem for which a position measurement is made on an electron via observing a photon of wavelength λ. Starting with an electron in a superposition of position states, Heisenberg states [7]:

> *... each position determination reduces the wave packet again to its original dimension λ.*

In the Copenhagen interpretation, a wave function that is initially in a superposition of the observable's eigenstates will generally change non-locally and discontinuously via a projection into the subspace of eigenstates corresponding to an eigenvalue of the observable when a measurement occurs. Such a change of the wave function is termed wave function reduction or wave function collapse.

An important possibility to consider occurs when a measurement device does not detect the presence of a particle. In this case, the result is negative measurement [195] or interaction-free measurement (IFM) [106]. Because the IFM subspace is not complete, IFM is considered a type of measurement that also involves a projection and will have an associated wave packet modification similar to reduction. The wave function in IFM is projected into the subspace that is orthogonal to the linear space spanned by the eigenstates corresponding to the device detecting the presence of the particle.

Discerning Wave Function Reduction

In terms of whether or not wave function reduction by itself can be justified to be a property of measurement versus unitary evolution is not immediately obvious. The original justification of the reduction in the paper by Heisenberg [7] uses the concept of the Heisenberg uncertainty principles. That is, if one reduces the position uncertainty of an electron through measurement, then its momentum uncertainty is increased. However, unitary evolution of the quantum wave function also respects the Heisenberg uncertainty principle. That is, suppose that through a unitary interaction with a device the position uncertainty as measured by the standard deviation of the electron's wave function was found to decrease, then one would expect the electron's momentum, which is related to the Fourier transform of the positional wave function, to similarly increase.

On the other hand, the Chapter 3 UMDT demonstrates that it is possible in principle based on the current quantum formalism, to distinguish unitary evolution from measurement. The Chapter 3 UMDT was based on a multiple degree-of-freedom reduction and not on a single degree-of-freedom reduction.

Consider a single degree of freedom of a particle that has undergone external orthogonalization such that under unitary evolution the state of system plus apparatus is given by $\sum_r c_r \,|\phi_r\rangle \otimes |\Psi_r\rangle$. The system is in the mixed state given by Equation (4.3) reproduced below for convenience:

$$\begin{pmatrix} |c_1|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |c_N|^2 \end{pmatrix}.$$

Suppose the final state of the particle under measurement occurs definitely as $|\phi_r\rangle$ for some r. One might argue that the state being definitely in some $|\phi_r\rangle$ is different than the mixed state of Equation (4.3). However, suppose an experiment prepares a set of K states by one of two methods. In Method 1, Alice prepares $|\phi_r\rangle$ using a random number generator that corresponds to probability $|c_r|^2$. This is repeated K times and the states deposited in a box. In Method 2, Alice prepares the mixed state of Equation (4.3) experimentally for each particle, and deposits K such states in a box. If the box is subsequently given to a second experimenter Bob, and he is asked to experimentally discriminate which of the two methods Alice used to prepare the states. Bob, however, finds that it is impossible based on current quantum theory to determine which method Alice used. Hence for a single degree of freedom, it does not appear that a measurement interpretation for which the outcomes definitely have occurred, can be experimentally falsified from a mixed state prediction. This has been referred to as the ignorance interpretation by Hughes [196]. A mixture of pure states that can be interpreted via the ignorance interpretation for which the state is in a pure state with some probability is termed a proper mixture. Mixtures that cannot be interpreted via the ignorance interpretation are termed improper [197, p. 58] by d'Espagnet. Generally, a subsystem that is part of a larger entangled state will result in an improper mixture because the state cannot be properly interpreted to exist as any particular pure state of which we are ignorant, and so a probability distribution is assigned over a set of pure states to represent the mixed state. If the latter ignorance interpretation were true, a hidden variable model for the local subsystems would exist and entangled states would not violate the Bell inequality. However, as such entangled states do violate the Bell inequality, it cannot be said that the mixed state of a subsystem of an entangled state actually exists as a pure state but that we are simply ignorant of it and thereby need to treat the matter probabilistically. On the contrary, no local probabilistic treatment suffices to explain the violation of Bell's inequality, and hence the ignorance interpretation of proper mixture absolutely fails and an improper mixture must be used to interpret the subsystems of entangled states.

On the other hand, the Chapter 3 UMDT demonstrates that it is possible to experimentally discriminate unitary evolution from measurement. However, the UMDT requires two measurements on separate degrees of freedom. The cases of measurement and unitary evolution now become experimentally distinguishable because the difference between the two cases can be manifested in a correlation difference between the measurements rather than a proper mixed state interpretation of the state. In this case the mixed state prediction of unitary evolution is an improper mixture. Wave function reduction, which is predicted to break the unitarily predicted entanglement under measurement, can result in an experimentally different result when multiple degrees of freedom are considered.