Bohr's Correspondence Principle

Bohr's theory was deliberately incomplete so that he could find principles that would allow a systematic search for a "rational generalization" of classical electrodynamics. A major conceptual tool that characterized Bohr's atomic work was the *correspondence principle*, his celebrated asymptotic consistency requirement between quantum theory and classical physics, that took various forms over the years and was often misunderstood by even his closest collaborators [312, p. 114], but guided his research and dominated quantum theory until the emergence of quantum mechanics in 1925-26 [419, p. 81]. It should be emphasized that the correspondence principle is not the elementary recognition that the results of classical and quantum physics converge in the limit $h \rightarrow 0$. Bohr said emphatically that this [420]

See the print edition of The Quantum Measurement Problem for quotation.

Instead, the correspondence principle is deeper and aims at the heart of the differences between quantum and classical physics. Even more than the previous fundamental work of Planck and Einstein on the quantum, Bohr's atomic work marked a decisive break with classical physics. This was primarily because from the second postulate of Bohr's theory there can be no relation between the light frequency (or color of the light) and the period of the electron, and thus the theory differed in principle from the classical picture. This disturbed many physicists at the time since experiments had confirmed the classical relation between the period of the waves and the electric currents. However, Bohr was able to show that if one considers increasingly larger orbits characterized by quantum number n, two neighboring orbits will have periods that approach a common value. And from the frequency of the light that is emitted in a jump between two such orbits according to Bohr's quantum postulate, one finds that it approaches the frequency of classical theory with this period. Therefore, even though the ***physics*** of the emission of light in quantum theory is quite different from that in the classical theory, the results approached approximate classicality with increasing n. Bohr's theory in a precise way contained within itself the results of classical theory and is a generalization of classicality in the sense of this correspondence principle.

Bohr first applied the correspondence arguments when he argued that as energy intervals $\Delta E = h\Delta\nu$ between stationary states become arbitrarily small the emitted frequencies would coincide with the frequencies expected from classical theory and he used this correspondence in 1913 to derive the energy levels of the hydrogen atom and subsequently in his analysis of the Stark effect. It could also function as a selection principle, as Bohr explained [421]:

> *Among the processes that are conceivable and that according to the quantum theory might occur in the atom we shall reject those whose occurrence cannot be regarded as consistent with a correspondence of the required nature.*
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> Niels Bohr, The Theory of Spectra and Atomic Constitution, Three Essays, Cambridge University Press, 1924.

Bohr's most careful statement of the correspondence principle appeared in a 1921 paper [422]. However, Bohr insisted that the correspondence principle was quantum mechanical and that its physical meaning did not depend directly on the validity of classical mechanics for the motion of an atomic system in stationary states. Bohr's statements following Heisenberg's development in 1925 of matrix mechanics, the first consistent theory of quantum mechanics, including his later complementarity interpretation of the wave-particle duality, continued to be influenced by the correspondence between the quantum and classical regimes. In Bohr's famous Como lecture of 1927 in which he first introduced the concept of complementarity, he talked of [310]

> *a far-reaching correspondence between the consequence of the classical theory and those of the quantum theory.*
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The correspondence principle influenced his interpretation of quantum mechanics and was a key element for his formulation of complementarity. Bohr felt that the principle had been fully incorporated into Heisenberg's matrix mechanics in that [198]

> *the whole apparatus of the quantum mechanics can be regarded as a precise formulation of the tendencies embodied in the correspondence principle.*
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Bohr's friend and colleague Georg von Hevesy (1885-1966) recounted Einstein's reaction to this aspect of Bohr's theory in a 1913 letter to Bohr [423, p. 113]:

> *I told him then that it is established now with certainty that the Pickering-Fowler spectrum belongs to Helium. When he heard this he was extremely astonished and told me: "Then the frequency of the light does not depend at all on the frequency of the electron? ... And this is an* ***enormous achievement****. The theory of Bohr must then be right."*
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> Andrew Whitaker, Einstein, Bohr and the Quantum Dilemma: From Quantum Theory to Quantum Information, Cambridge University Press 2006.

Einstein later summarized Bohr's achievements in atomic theory [339]:

> *That this insecure and contradictory foundation [of quantum physics in those years] was sufficient to enable a man of Bohr's unique instinct and tact to discover the major laws of the spectral lines and of the electron shells of the atoms together with their significance for chemistry appeared to me like a miracle—and appears to me a miracle even today.*
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> Albert Einstein, Autobiographical Notes, In: Albert Einstein: Philosopher-Scientist, Paul Arthur Schillp (Editor), Cambridge University Press 1949.