Planck's Fortunate Guess

In response to improved experimental measurements, Planck had initially guessed Equation (5.3) in October 1900 using thermodynamic entropy arguments to interpolate two expressions: first, a heuristic formula of Rayleigh's that accurately fit experiments at low frequencies. In this regime, it is equivalent to the improved expression of Rayleigh-Jeans which was derived later, given by:

$$B^{RJ}(\nu, \mathrm{T}) = \frac{8\pi}{c^3}\nu^2 \mathrm{kT}, \tag{5.5}$$

and secondly, a formula proposed by Wilhelm Wien (1864-1928) that accurately fit experiments at high frequencies:

$$B^{W}(\nu, \mathrm{T}) = \frac{8\pi a}{c^3}\nu^3 \exp(-b\nu\mathrm{T}). \tag{5.6}$$

Here the universal constants a and b are of course precursors of combinations of k and h. By introducing the new constant h, Planck used an entropy argument to successfully find an interpolation between the two laws yielding Equation (5.3). He described his expression as a "fortunate guess," but it proved to be in perfect agreement with all the experiments to date and would continue to agree with continually refined future experiments. Today this includes the thermal radiation of the cosmic microwave background left over from the Big Bang of cosmology. It has been said of Planck's interpolation that [410, p. 18]:

> *See the print edition of The Quantum Measurement Problem for quotation.*

By way of his "vehement obstinacy" in obtaining this law, Planck could be said to have reached a "conflagration of clear sight" [411]. Ineluctable modality of the quantum. The auxiliary variable Planck had needed for his interpolation was called "*h*" after the German word *hilfsgrösse* [412] which translates to "auxiliary variable," humble nomenclature for a quantity with grand implications. Such is how history is made. The fortunate guess was later shown to be exact within quantum mechanics.

However, at this point there was not yet any quantization in Planck's work despite the appearance of h. Although not appreciated at the time, there was, however, foreshadowing of the quantum wave-particle duality at this stage. Planck had followed closely the works of Rudolf Clausius (1822-1888), one of the founders of thermodynamics who had formulated the Second Law of Thermodynamics in terms of the irreversible increase of entropy. The entropy S was a new quantity introduced by Clausius as a measure of how much thermal energy is unavailable for conversion into mechanical work. Planck had taken the second law to be absolute and that entropy would increase without exception and he was skeptical of the atomic hypothesis that all matter is composed of indivisible particles. In contrast, the pioneers of statistical mechanics viewed the world in term of atoms: Maxwell, Ludwig Boltzmann (1844-1906), and Josiah Willard Gibbs (1839-1903). Planck had spent the previous several years in attempts to understand the radiation problem from first principles without success and had reached the point of taking the desperate steps of using Boltzmann's statistical mechanical methods.

Since Kirchhoff had shown that thermal radiation was independent of the particulars of the radiating body, Planck chose the simplest model of a set of electrically charged oscillators with energy U (one for each frequency) filling a cavity with reflecting walls. As summarized in Appendix 5.A, this led him to guess an interpolation expression, $\alpha U + \beta U^2$, which exactly echoes the first identification of wave-particle duality later found by Einstein in 1909. Einstein had applied his previously derived formula for the fluctuation in energy to Planck's law and found:

$$\Delta E^2 = h\nu < E > + \frac{c^3}{8\pi\nu^2} < E >^2. \tag{5.7}$$

This is given per unit volume and frequency range and $< E > = U$ is the average oscillator energy. The partition into the linear and quadratic terms in Equation (5.7) just matches that for Planck's fortunate guess for his interpolation and explains why Planck's resulting radiation law Equation (5.3) exactly matches that later found from quantum mechanics.

Einstein immediately recognized the linear term in Equation (5.7), corresponding to the Wien limit, as having a particle origin corresponding to the fluctuations of a gas of independent particles. The quadratic term, corresponding to Rayleigh-Jeans, could be identified with fluctuations of the superposition of random standing waves in a small cavity. What had emerged was apparently a type of "wave-particle" duality but it was perplexing to Einstein in how to think of this in terms of his previous 1905 proposal of light-quanta, now called *photons*, as independent localized particles. In a 1909 letter to Lorentz, Einstein said [413]:

See the print edition of The Quantum Measurement Problem for quotation.

As will be seen, this moment is the onset of Einstein's continuing difficulties in facing the issues of wave-particle duality and later entanglement, even though Schrödinger did not coin that term until 1935 in relation to the measurement problem. Although not then recognized in full, wave-particle duality goes hand-in-hand with entanglement and the implications of this can already be seen emerging in 1909, with much more to come in the story. One piece of the puzzle was that the independent light-quanta had to have the additional quantum property of *indistinguishability*, which affects the counting of the possible energy states they may occupy. And this property implies that they are actually not quite independent but in a subtle way, the details of which would have to wait for the discovery of Bose-Einstein statistics in 1924. However, this is what allowed the Wien limit at low temperature to be handled in terms of "classical" independent particles while the explicit appearance of h counterintuitively appears at high temperatures in the Rayleigh-Jeans limit.

Therefore, we see in hindsight that Planck's fortunate guess already contained the seeds of wave-particle duality. But Planck did not yet have a deep understanding of the physics that lie behind the radiation law that had emerged from his guess. With the new interpolation expression Equation (5.3) now in hand, he then spent eight strenuous weeks between October and the December meeting of the German Physical Society searching for a theoretical understanding of it. In his derivation, Planck reluctantly drew upon Austrian physicist Ludwig Boltzmann's 1877 result for the increase of entropy utilizing a relation between entropy S and probability W, summarized by the equation $S = k \log W$. Although Planck had previously wanted to avoid the statistical conception of irreversibility advocated by Boltzmann, he reinterpreted the probability in Boltzmann's relation as a measure of the elementary disorder depending on the internal structure of his cavity oscillators. Both Planck's and Boltzmann's investigations were of the evaluation of the maximum entropy, the most probable equilibrium states of systems with energies in random distributions. In order to calculate the probability of a state with an energy shared among many oscillators of the same frequency, it was essential that Planck treat the energy as being composed of finite magnitude cells of size E in phase space. In Boltzmann's derivations, the size of the cells, representing molecules with different velocities, were arbitrary and could be taken to zero, while Planck could arrive at a derivation of Equation (5.3) only with finite cells of blackbody cavity oscillators with different frequencies, proportional to multiples of $E = h\nu$ (see Appendix 5.A).

Planck described his plight in taking the step of discrete energies in a letter to R.W. Wood thirty years later, describing it as "an act of desperation" [414]:

> *"I knew then that the problem is of fundamental significance for physics; I knew the formula that reproduces the energy distribution in the normal spectrum; a theoretical interpretation* ***had*** *to be found at any cost, no matter how high."*

Max Planck's discovery of the quantum in 1900, which combined relentless trial and error on Planck's part using the newest available ideas from statistical mechanics along with his awareness of the results emerging from the precision black-body developed at the Physikalisch-Technische Reichsanstalt nearby after he had moved to Berlin [415],

See the print edition of The Quantum Measurement Problem for quotation.

As seen in the section *The Quantum Triumvirate*, Planck's quantum of action led to the rise of nondeterminism and the insights for how the triumvirate of nondeterminism, entanglement and nonlocality shape quantum theory. From the beginning, Planck also understood its relation to atomism and their spectral lines [416]:

See the print edition of The Quantum Measurement Problem for quotation.

Over the next decade, scientists gradually came to terms with the implications of Planck's result. There was little immediate interest up to 1905 when Einstein took the next step of introducing the light-quanta. As discussed in the section *Atomism Prevails*, he was also resisted by those who opposed the atomic viewpoint: Ernst Mach (1838-1916), Wilhelm Ostwald (1853-1932), and Pierre Duhem (1861-1916) who had also battled against Boltzmann. Some historians have argued whether or not Planck actually accepted the discontinuous aspect of his derivation both from the historical record of Planck's work leading up to the derivation and his subsequent statements seemingly hedging on the issue [417] [418]. The truth may lie somewhere in between since in his Nobel Prize address in 1920, Planck credits Einstein with taking the decisive step:

> *The failure of all attempts…. soon left no doubt: either the quantum of action was a fictitious quantity, or the derivation of the radiation law rested on a truly physical thought…Experiments have decided in favor of the second alternative. But science does not owe the prompt and indubitable character of this decision to tests of the law of the energy distribution of thermal radiation, and even less to my special derivation of this law; it owes that to the unceasing progress of the researchers who have put the quantum of action to the service of their investigations. A. Einstein made the first breakthrough in this domain.*

Planck was clearly the discoverer of the quantum although subsequent researchers uncovered the physical meaning of it, and foremost was Einstein. Einstein's influence stands at the crossroads of space-time and quanta. Shown in Figure 5.8 are trajectories in space-time with the constraints imposed by relativity as well as apparently similar trajectories originating from quanta, the spherical quantum wave functions of alpha particles. Understanding the relation between space-time and quanta occupied the first decades of the 20th century.