# From EPR to the Present

## *The Quantum Triumvirate*

Bohr's response to the 1935 Einstein-Podolsky-Rosen (EPR) paper [474] was essentially the same as his view of Laplace's Demon. The EPR argument involves *the quantum triumvirate* of non-determinacy, entanglement and nonlocality. EPR considered an entangled state of two widely separated systems *A* and *B*, and considered how *A* is affected by a remote measurement on *B*. EPR used *locality* as a criterion to argue that the value of a local observable must be definite and not affected by a measurement at the remote location and gave a sufficient criterion for an *element of reality* of a physical quantity [474]:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.*
>
> A. Einstein, B. Podolsky, and N. Rosen, Physical Review 47, 777 (1935), Copyright (1935) by the American Physical Society. https://doi.org/10.1103/PhysRev.47.777

EPR demonstrated that the non-commuting variables of position and momentum could be measured more accurately than allowed by the uncertainty principle unless the measurement of the remote particle instantaneously affects the local particle. They concluded that quantum theory did not account for all elements of reality and must be incomplete. EPR expected that the inclusion of local hidden variables would make quantum theory complete and allow definite values to be given for all physical variables. Einstein later described the argument as forcing us to choose between the following two assertions [339]:

> (1) the description by means of the psi-function is complete.
>
> (2) the real states of spatially separate objects are independent of each other.

Bohr's response [469] focused on the ambiguity of EPR's *criterion of reality* and emphasized the different experimental arrangements required for unambiguously addressing complementary physical variables in quantum theory,

> *In fact, the renunciation in each experimental arrangement of the one or the other of two aspects of the description of physical phenomena—the combination of which characterizes the method of classical physics, and which therefore in this sense may be considered as complementary to one another—depends essentially on the impossibility, in the field of quantum theory, of accurately controlling the reaction of the object on the measuring instruments, i.e., the transfer of momentum in case of position measurements, and*

> *the displacement in case of momentum measurements.*
>
> N. Bohr, Physical Review 48, 696 (1935), Copyright (1935) by the American Physical Society.
> https://doi.org/10.1103/PhysRev.48.696

Experimental tests of Bell's inequality eventually showed that quantum nonlocality implies that the local hidden variables sought by EPR do not exist. Bell's results implied that locality and realism in the sense of EPR are incompatible with quantum mechanics. The demands of no-signaling between space-like separated systems as well as a condition of locality imply that correlations comply with Bell's inequalities. However, the inequalities are violated by correlated entangled quantum particles. No-signaling requires that instantaneous communication is impossible. If no-signaling is accepted, Bell measurements cannot be deterministic, and hence they are random as long as the measurement settings are freely chosen. Nonlocal correlations cannot be revealed only by local descriptions of the systems and this instead requires an entangled state. Local observations will then generally reflect an intrinsic randomness in the presence of nonlocal correlations even if the entangled state is pure and completely known. Greenberger, Horne, and Zeilinger extended the EPR entangled states to three particles [568], which have additional subtle properties allowing an illustration of Bell's theorem in a very direct way, as shown in Appendix 5.A.

Some thirty years after EPR, a further result, the Kochen-Specker theorem [81] [569], reinforced the argument for nondeterminism in a different way than EPR by showing (in Hilbert space dimensions $d \geq 3$) that if we want to add hidden variables to avoid nondeterminism, the hidden variables have to be *contextual*; i.e., the outcome depends on the specific experimental arrangement used to measure the observable. Kochen-Specker does not refer to nonlocality but rather to the incompatibility of hidden variables associated with the system being measured. This incompatibility addresses the mutual exclusiveness of experimental arrangements and further strengthens Bohr's complementarity principle some thirty years earlier as a type of contexuality argument.

The tension between the concepts of non-determinacy, entanglement, and nonlocality also impacts how the quantum state or wave function is viewed. The question of the meaning of the wave function goes back to the beginning of quantum theory, with de Broglie and Schrödinger initially arguing that it was a real physical wave [4]. After Born's introduction of indeterminacy, the Copenhagen interpretation of Bohr, Heisenberg, and Pauli viewed the wave function as a wave of probability amplitude. Heisenberg further suggested that the quantum probability amplitude waves can be interpreted as a quantitative version of the concept of *potentia* from Aristotle's philosophy [552, pp. 9-10]. The tendency for an event to take place was recognized as an aspect of the world having an intermediate reality between object and idea. Potentiality and actuality were a dichotomy used by Aristotle to analyze a range of issues of the physical world related to motion, causality and physiology. Similarly, Heisenberg suggested that the quantum laws of nature determine the possibility of occurrence and not the phenomenon itself.

As discussed in Chapter 4, some recent interpretations and models for quantum theory such as many-worlds, de Broglie-Bohm, and spontaneous collapse regard the wave function as real, and are known as $\psi$*-ontic* theories (a state of reality). Other interpretations such as quantum Bayesian and quantum information-based approaches relate the wave function to representations of knowledge and are known as $\psi$*-epistemic* theories (a state of knowledge). More recently, Pusey, Barrett, and Rudolph (PBR) [570] proved a theorem showing that the quantum state must be ontic in a wide class of quantum theories. The main underlying assumption of the PBR theorem is that composite systems prepared in a product state should be independent of one another; i.e., the ontic distribution for $|\psi_a\rangle\otimes|\psi_b\rangle$ is the product of the ontic distribution for $|\psi_a\rangle$ and the ontic distribution for $|\psi_b\rangle$. Much effort has been spent on examining a variety of assumptions and approaches to determine under what conditions the wave function is allowed to be considered either ontic or epistemic [571].

However, a complete picture of the subtle roles played by indeterminacy, entanglement and nonlocality and how they are interrelated has emerged most fully from an understanding of a variety of Bell inequality experiments, beginning in the 1960s and up to the present day. For classical theories, randomness cannot be intrinsic but only a result of an incomplete description of the system. Quantum theory is known to give probabilistic predictions for particular experiments in which the preparation of the system is essentially perfect. Einstein claimed in the EPR paper that this could be explained by the incompleteness of quantum mechanics, and there should be a complete theory giving a deterministic result for every experiment. However, as discussed in Chapter 3, Bell's theorem indicated that local hidden variable theories are inconsistent with quantum mechanics. In hidden-variable theories with a local causal structure, correlations between space-like separated measurement events satisfy a variety of Bell inequalities. Numerous quantum systems have by now been experimentally shown to violate Bell inequalities. However, there are subtleties among the assumptions made in order to conclude that intrinsic nondeterminism in nature follows from quantum nonlocality. If no-signaling had not been assumed, then it would be possible to have deterministic descriptions of quantum correlations.

If no-signaling is assumed, the conclusion that Bell measurements imply nondeterminism relies on the further assumption that the Bell measurement settings are freely chosen and truly random. The theorem does not apply unless one has freedom to choose the detector's settings without modifying the state that is to be measured. However, it is never possible to completely certify the randomness of the settings and rule out scenarios, however implausible, to explain the initial random settings. An extreme example is *superdeterminism*, so-named by Bell as a possible loophole for his theorem, which hypothesizes extravagantly that all processes are completely predetermined by conditions in the past light-cone of both observers, so that each space-time point encodes the initial state of the universe. As Shimony, Horne, and Clauser had summarized [572]:

> *In any scientific experiment in which two or more variables are supposed to be randomly selected, one can always conjecture that some factor in the overlap of the backward light cones has controlled the presumably random choices. But, we maintain, skepticism of this*

> *sort will essentially dismiss all results of scientific experimentation. Unless we proceed under the assumption that hidden conspiracies of this sort do not occur, we have abandoned in advance the whole enterprise of discovering the laws of nature by experimentation.*
>
> Search for a Naturalistic World View, Volume II, Natural Science and Metaphysics, Abner Shimony, Cambridge University Press.

The implication is that if the initial conditions were to be somewhat different, the entire quantum mechanical theory would dissolve and not describe the full range of phenomena currently accounted for by quantum mechanics [573].

Though logically impossible to rule out, such conspiracy-type models become even further far fetched by other developments, which highlight the tension between quantum theory and determinism in a particularly striking way. These are methods for *Bell-certified randomness*, including quantum randomness expansion (generating and certifying a large number of random bits from a small seed of random bits) and quantum randomness amplification (generating uniformly random bits called *free* randomness from imperfectly random ones). These experimentally demonstrated protocols for certification, expansion and amplification use Bell measurements to produce outcomes that are fully unpredictable and achieve tasks that are not possible classically [574] [575] [576] [577] [578] [579]. For example, the scaling of an initial seed randomness has been demonstrated for protocols that give an exponential expansion of the randomness as strong as exponential, so that a seed of n-bits grows to $2^n$-bits. The *Bell-certified* means that the resulting bits are close to uniformly random in terms of a quantitative relation between the amount of the Bell-inequality violation and the randomness observed that is also independent of the details of the measurement devices, i.e., *device-independent quantum randomness*. This is the strongest certification of nondeterminism that can be expected using quantum nonlocality. In a sense, this pushes any alternative possibilities of conspiracies in the measurement settings as tightly as possible into a corner so that they would have to be truly outlandish to be functioning in our universe. Such protocols also allow devising random number generators for which a Bell inequality violation guarantees that the output is random and immune to outside adversaries.

Although intrinsic quantum randomness can be certified in terms of nonlocality and entanglement, the relationships among the quantum triumvirate of nondeterminism, entanglement, and nonlocality are subtle and not as direct as might be expected; not simply, e.g., less Bell violation $\Longrightarrow$ less quantum randomness [580]. Instead, a maximum quantum randomness can be certified from arbitrarily small amounts of nonlocality or entanglement. The amount of randomness certified by nonlocal quantum correlations is inequivalent both to entanglement and nonlocality even though nonlocality is necessary for entanglement and nonlocality is necessary for certifying randomness. Probability distributions with maximal nonlocality do not necessarily contain maximal randomness. And contrariwise, distributions with arbitrarily small amounts of nonlocality may contain nearly maximal randomness. The quantitative relationships between these quantities require the challenging task of characterizing extremal properties of the boundaries of quantum correlations. As shown in [580], in the type of CHSH plot familiar from Chapter 3, the region of maximal randomness can be arbitrarily close to the CHSH bound of $S \leq 2$, below which local hidden variable models become possible. Violations of CHSH had been discussed in Chapter 3 as a crucial element in devising UMDT tests for distinguishing measurement from unitary evolution in terms of CHSH bounds. The generation of maximal certified quantum randomness, the signature of intrinsic nondeterminism, is intriguingly seen to emerge near this borderline of nearly product states with arbitrarily small entanglement.

The relation between the constraint of no-signaling and the randomly generated response to measurement exemplified by the Bell theorems was further extended by Conway (known outside of academia as inventor of the celebrated cellular automaton, *Game of Life*) and Kochen (of the Kochen-Specker Theorem) in results they brazenly called the *Free Will Theorem* (FWT) and a strengthened version, the *Strong Free Will Theorem* [581] [582], which led to exchanges in the literature. These results in effect claimed that, in the absence of signaling, if Alice and Bob have the free will to measure their particles, then the particles have their own free will for how to respond. These results combined elements of EPR and Kochen-Specker along with the relativistic consequence of there being no preferred frame of reference determining the order of Alice and Bob's freely chosen space-like separated measurements to conclude that their measurements also cannot be determined by the prior state of the universe. Although Alice and Bob could be replaced by (pseudo) random number generators, Conway and Kochen contend that free will would still be required to choose the random number generators to avoid the possibility of them having been predetermined in the past. However, the use of the term *free will* here need not be quite the strong free will that haunts the psyche of philosophers. It could as well be the *free randomness* generated by the protocol of randomness amplification discussed previously [575]. With randomness amplification, sufficiently strong quantum correlations ensure that even if Alice and Bob cannot choose the measurements perfectly freely, the generated outputs are nonetheless perfectly free randomness. It has also been shown that with no-signaling and arbitrary freedom of choice of measurement settings, there are quantum processes with fully intrinsic randomness that are not mixed with apparent randomness due to the incompleteness of quantum mechanics [583]. In those cases, no alternative theories based on either signaling or no freedom of choice could give better predictions than quantum theory.