The Rise of the Measurement Problem

The Characteristic Trait

By 1935, Schrödinger had begun to return to his earlier views which had argued against discontinuous collapse. As a result, he wrote a series of three papers in 1935-36 [156] [119] [464] that appeared just months after the EPR paper and first began introducing the concept and properties of entanglement. The initial paper, *The Present Status of Quantum Mechanics*, also introduced within a discussion of the measurement problem the hypothetical dilemma of the living and dead cat, *Schrödinger's Cat* (discussed in Chapters 1 and 2), with the consequence that there was apparently nothing to prevent microscopic oddities from appearing in our macroscopic world [119]. In these papers Schrödinger coined the term *verschränkung*, now ubiquitously translated into English as *entanglement*, that he described as *the characteristic trait* of quantum mechanics in the paper *Discussion of Probability Relations between Separated Systems* [156],

> *When two systems of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described the same way as before, viz., by endowing each of them with a representative state of its own. I would not call that one but the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled.*
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> E. Schrödinger, Discussion of Probability between Separated Systems Proceedings of the Cambridge Physical Society 31 (4), 555-563 (1935).

Schrödinger's papers were motivated partly by his correspondence with Einstein and by the 1935 Einstein-Podolsky-Rosen (EPR) paper, which had argued that quantum mechanics is incomplete. By this time, Einstein was at the Institute for Advanced Study (IAS) in Princeton, New Jersey, having fled to America from the turmoil in Germany [465]. Einstein's assistant, Nathan Rosen, had recently written a paper presenting the first reliable calculation of the structure of the hydrogen molecule. This turned out to require wave-functions which could not be written as products of wave-functions for each of the two electrons in the molecule. These were entangled states. At the traditional 3 o'clock IAS tea, Rosen described their properties, and Einstein immediately saw the implications for the interpretation of quantum mechanics. They were joined by Boris Podolsky who later also proposed writing an article. However, the subsequent events quickly became exasperating for Einstein. The *New York Times* headline of May 4, 1935 read, "Einstein Attacks Quantum Theory" after Podolsky leaked the development to the press. Einstein was quite irritated at Podolsky for doing this and avoided him thereafter, telling the *Times* that the information "was given to you without my authority." Podolsky had also written the article and Einstein was not pleased with the result, writing to Schrödinger "For reasons of language, this was written by Podolsky after several discussions. Still, it did not come out as well as I had originally wanted; rather, the essential thing was, so to speak, smothered by the formalism [*gelehrsamkeit*]." [465] Behind the scenes, Einstein often clarified his thoughts in letters which sometimes give more insight than the journal articles. In an August 8, 1935, letter to Schrödinger, Einstein attempts a simpler incompleteness argument by considering a macroscopic example to avoid having to assume locality as was done in the EPR paper [466, p. 78]:

> *The system is a substance in chemically unstable equilibrium, perhaps a charge of gunpowder that, by means of intrinsic forces, can spontaneously combust, and where the average life span of the whole setup is a year. In principle, this can quite easily be represented quantum-mechanically. In the beginning the psi-function characterizes a reasonably well-defined macroscopic state. But, according to your equation, after the course of a year this is no longer the case. Rather, the psi-function then describes a sort of blend of not-yet and already-exploded systems. Through no art of interpretation can this psi-function be turned into an adequate description of a real state of affairs; in reality, there is no intermediary between exploded and not-exploded.*
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Schrödinger's letter in reply of September 19, 1935, is the first appearance of his thought-experiment of the living and dead cat, telling Einstein that he had constructed an example very similar to his exploding powder keg, thus suggesting the origin of *Schrödinger's Cat*. However, Einstein was pleased with this extension to a cat, as in this 1950 letter to Schrödinger, though he sometimes conflated Schrödinger's use of cyanide with his use of gunpowder for the demise of the cat,

> *You are the only contemporary physicist, besides Laue, who sees that one cannot get around the assumption of reality—if only one is honest…They sometimes believe that the quantum theory provides a description of reality, and even a complet**e** description; this interpretation is, however, refuted, most elegantly by your system of radioactive atom + Geiger counter + amplifier + charge of gun powder + cat in box, in which the ψ-function of the system contains the cat both alive and blown to bits.*
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Einstein had also remarked that de Broglie had "lifted a corner of the great veil" [467] and it was de Broglie's dissertation that had led Schrödinger to wave mechanics. With the presence of entanglement, Schrödinger could emphasize the wave-function viewpoint and instead view measurement as concerning both elements of the entangled system and detector. In this way, there would be no need to refer to particles before an observation. The act of measurement would then be responsible for the collapse into a discrete value [451]. In his 1935-36 entanglement papers, Schrödinger examined a composite system in Hilbert space $H_1 \otimes H_2$ describing systems S_1 and S_2 respectively. He showed that any state vector can be written as the essentially unique biorthogonal decomposition,

$$\psi(x,y) = \sum_k a_k g_k(x) f_k(y)$$

with complete sets of unit vectors $\{g_k\}$ and $\{f_k\}$, which comprise eigenvectors with eigenvalues λ_k^g and λ_k^f of some observables of the systems S_1 and S_2. Measurements in the y-system with eigenfunctions $f_k(y)$ with different eigenvalues occur with probability $p_k = |a_k|^2$, in which case it follows that the x-system must be assigned the wave function $g_k(x)$ in all cases. Schrödinger thus found the troubling result that instantaneous changes in the state of system S_1 result from measurement on the spatially distant system S_2. Therefore, the Copenhagen interpretation of measurement is nonlocal due to entanglement. This includes the special case of states considered by EPR in which all the $|a_k|^2$ are equal so that every observable of one system is determined by an observable of the other one. The EPR argument used a special perfectly correlated state of particles with equal and opposite momenta that we now recognize as entangled,

$$|\psi(x)\rangle_{EPR} = \int dq\, |q - x\rangle\, |q\rangle = \int dp\, e^{ipx} |-p\rangle\, |p\rangle$$

Bohm's version of EPR used two spin-1/2 particles or qubits in the superposition state [468],

$$|\psi_{AB}\rangle_{EPR} = (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)/\sqrt{2}\,.$$

EPR had argued that this superposition together with the quantum measurement postulate leads to situations in apparent contradiction to the causality principle of relativity since a measurement by Alice in the $\{|0\rangle, |1\rangle\}$ basis will lead to Bob's spatially separated state to collapse into the state $|0\rangle$ or $|1\rangle$ corresponding to the measurement outcome of Alice. Causality would appear to be violated as if under the influence of a *spooky action at a distance.* In Bohr's reply to EPR, he invoked his concept of complementarity to explain that this state of affairs is not problematic [469]:

> *There is no question of a mechanical disturbance of the system under investigation [in the EPR situation] ... there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the*

> *system.*
>
> N. Bohr, Physical Review 48, 696 (1935), Copyright (1935) by the American Physical Society. https://doi.org/10.1103/PhysRev.48.696

Thus, Alice and Bob end up with a perfectly correlated classical bit after the experiment with no transfer of information. The consequences of the possibility of entanglement becoming degraded at large separations had been considered by Schrödinger and also by Wendell Furry [470] [471] so that entangled states spontaneously localize to mixed product states. Schrödinger had considered this as a way to avoid the “paradox,” whereas Furry agreed with Bohr and used the example to show that any such modified version of quantum mechanics must immediately be viewed as wrong. During the period 1960-70’s when Clauser was developing his photon anti-bunching experiments, a group led by Edwin Jaynes (1922-1998) had proposed to the quantum optics community a *neoclassical theory* (NCT) of light-matter interactions and challenged them to disprove it with evidence [461]. NCT assumed that an atom’s wave-function is governed by Schrödinger’s equation while the radiation fields are described by the classical Maxwell’s equations. This might be viewed as a modern version of Schrödinger’s classical field description of Compton scattering, although NCT was more sophisticated and included effects such as radiation reaction. In a sense, it may even hark back to Planck, with classical fields and quantized energy exchange. Jaynes et al. were able to predict an impressive range of phenomena using NCT including absorption of radiation, spontaneous emission of radiation, the Lamb shift and the black-body radiation spectrum. However, Clauser pointed out that when applied to EPR correlations, NCT would exhibit an example of the Schrödinger-Furry hypothesis with vanishing entanglement at large separations. The Schrödinger-Furry hypothesis was known to be false by that time so Jaynes conceded that NCT had been refuted. Of course, it would also be refuted by the full power of the Bell inequality experiments.

In his 1936 paper, Schrödinger had furthermore demonstrated that unitary transformations, $f_k(y) = \sum_i \alpha_{lk}\, h_i(y)$, can be found for the eigenstates of the y-system so that the state $\psi(x,y)$ can be shown to take the form

$$\psi(x,y) = \sum_i \sqrt{w_i}\left(\sum_k \frac{\alpha_{ik}\sqrt{p_k}}{\sqrt{w_i}}\, g_k(x)\right) h_i(y)$$

where $w_i = \sum_l p_l |\alpha_{il}|^2$. As a consequence, an appropriate observable corresponding to the eigenfunction $h_i(y)$ can be found to be measured on system S_2 so that one can transform the state of S_2 into *any* state of $S_1$with probability $|w_i|$. Such a process is called *steering,* which has received increasing recent attention from a quantum information viewpoint. In some sense, steering is an even spookier action, giving Alice the ability to affect Bob’s state through her choice of measurement basis. EPR is merely a particular case of steering. As Schrödinger put it [464],

> *It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the*

> *experimenter's mercy in spite of his having no access to it.*
>
> E. Schrödinger, Probability relations between separated systems, Proceedings of the Cambridge Physical Society 32, 446 (1936).

In the hierarchy of nonlocality, it is now known that steerable states are a subset of entangled states and a superset of states that can exhibit a Bell nonlocality [189]. Quantum steering gives a unified description of entanglement for pure bipartite systems with applications in quantum teleportation, entanglement monotones, quantum cryptography and Bell inequalities.

One month after the EPR paper was published, another paper appeared by Einstein and Rosen (ER), called *The Particle Problem in General Relativity* [472]. Dissatisfied with quantum mechanics, Einstein was interested in the extent to which the field theory method of his theory of gravity, General Relativity, could be used to account for atomic phenomena. Einstein sought an approach based on field theory in such a way as to avoid singularities so that the masses of particles are not concentrated at a point. Einstein thought this would ultimately be the real meaning of Heisenberg's uncertainty principle. In the ER paper, the question is asked:

> *Is an atomistic theory of matter and electricity conceivable which, while excluding singularities in the field, makes use of no other field variables than those of the gravitational field and those of the electromagnetic field in the sense of Maxwell?*
>
> A. Einstein and N. Rosen, Physical Review 48, 73 (1935), Copyright (1935) by the American Physical Society. https://doi.org/10.1103/PhysRev.48.73

The ER paper presented a solution of General Relativity in the form of what is now called an *Einstein-Rosen bridge*, a space-time construction that makes a smooth tube-like connection or bridge between two distinct pieces of space-time. The ER-bridge is now often also called a *wormhole,* as coined by John Wheeler. Einstein was not ultimately successful in finding manifestations of quantum mechanics from field theory. However, there have been recent attempts to relate the ER solution with actual EPR correlations and thus bring entanglement into the picture with gravitational field theory solutions. General relativity contains solutions in which two distant black holes are connected through the interior by ER-bridges. Although the black holes might be spatially distant, General Relativity shows that the ER-bridge cannot be used to transmit information between them. For the purposes of resolving issues in black-hole physics, Maldacena and Susskind postulated that the ER-bridges between two black holes are always created by EPR-correlations between the microstates of the two black holes, calling the proposed relation *ER=EPR* [473]. Namely, quantum entangled particles in an EPR state are connected by an ER-bridge. Just as EPR states cannot be used to signal acausally, neither can ER-bridges because they are not traversable. An ER=EPR relation suggests there would also be a quantum mechanical version of a classical ER-bridge that supports quantum nonlocality. If valid, something like ER=EPR might finally achieve a relation via entanglement between the atomistic and gravity domains sought by Einstein in 1935, although almost certainly not in a way that Einstein would have approved.

Howard summarizes the argument by Einstein against the completeness of quantum mechanics [3] which had not been presented as clearly in the 1935 Einstein-Podolsky-Rosen (EPR) paper [474],

> *See the print edition of The Quantum Measurement Problem for quotation.*

The first entangled state generated within quantum mechanics may have been from Born's 1926 paper *On the Quantum Mechanics of Collisions*, in which he introduced the probabilistic interpretation of quantum mechanics [6]. Born carried out a calculation of electron-atomic scattering where the system is initially in a product,

$$\psi^0_{n,\tau}(q_k, z) = \psi^0_n(q_k)\sin(2\pi z/\lambda).$$

The electron is in a plane wave momentum eigenstate incident on the atom and the atom is in an energy eigenstate. Thus, there is initially no entanglement between electron and atom. After the interaction, the state after the scattering was calculated using a perturbation method with the result clearly in the form of an entangled state in terms of momentum eigenstates scattered in different directions and excited atomic states,

$$\psi^1_{n,\tau}(x, y, z; q_k) = \sum_m \iint d\omega\ \Phi_{n,m}(\alpha, \beta, \gamma)\sin k_{n,m}(\alpha x + \beta y + \gamma z + \delta)\psi^0_m(q_k).$$

Born then makes this conclusion about the result,

> *If one translates this result into terms of particles, only one interpretion is possible.* $\Phi_{n,m}(\alpha,\beta,\gamma)$ *gives the probability* for the electron, arriving from the z-direction, to be thrown out into the direction designated by the angles* α, β, γ*, with the phase change* δ*...Schrödinger's quantum mechanics therefore gives quite a definite answer to the question of the effect of the collision; but there is no question of any causal description. One gets no answer to the question, "what is the state after the collision," but only to the question, "how probable is a specified outcome of the collision."*

The asterix refers to the famous footnote saying that the probability is actually given by the square of $\Phi_{n,m}$ (though it should actually have said "absolute square"). Born is led to hypothesize indetermism in quantum mechanics from the terms of an entangled state decomposition, though how this entanglement would depend on the basis of expansion was of course not yet appreciated in 1926.

Following Born's pioneering work, wave-mechanical solutions for collisions involving entangled states were presented by C. G. Darwin in the paper "A collision problem in wave mechanics," [475] and building on Darwin's techniques, N.F. Mott explained the tracks of α-particles in cloud chambers in the paper "The wave mechanics of α-ray tracks" [428] as discussed in the section *Einstein's Quandary*.

According to Darwin,

> *Before the first collision, (the wave function) can be represented as the product of a spherical wave for the α particle, by a set of more or less stationary waves for the atoms…[The] first collision changes this product into a function in which the two types of coordinates are inextricably mixed.*
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The calculations of Darwin and Mott clearly involved entangled states of the entire system. Heisenberg also analyzed the cloud chamber problem in his 1929 University of Chicago lectures but took a more flexible approach for how to divide the compound structure into system and the observational device. For Heisenberg, the arbitrariness in switching from a corpuscular representation to a wave representation whenever convenient reflected the freedom in choice of the *cut* (*Schnitt),* as Heisenberg called it, between system and device [426].

Collisions could evidently produce complicated entangled states. However, a more systematic understanding of entanglement would benefit from the experimental realization of particular entangled states, especially in the form of EPR correlations. Production of the first EPR states would have to wait for the ability to control photon polarization states. The methods for doing this were analyzed in the decade following the EPR paper, during the period of 1947-1949 [476]. Dirac's ideas on particle-pair production in 1930 [477] were discussed by John Wheeler in 1946 [478] and the detailed quantum physics of the process and a rendition of the experiment by Pryce and Ward appeared in 1947 [479] with all the elements required for quantum entanglement, Figure 5.16.

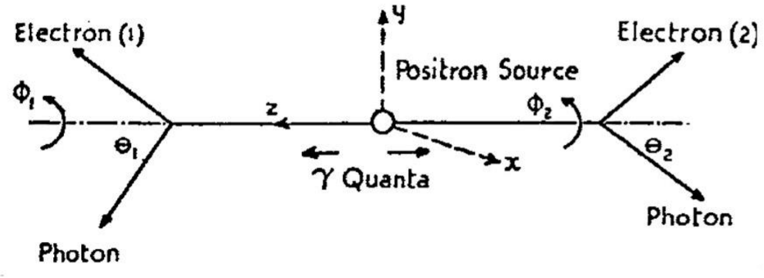

Figure 5.16: Proposal from 1947 of the method which led to the first realization of polarization entangled photons, i.e. EPR correlations [479].
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https://doi.org/10.1038/160435a0

The theory was independently derived by Snyder et al. in 1948 [480]. Two groups published experimental efforts by 1948 [481] [482] and more definitive results were reported by Wu and Shaknov in 1950 [483]. This was the same Chien-Shiung Wu (1912-1997) who would become prominent for her 1956 experimental confirmation of the violation of parity conservation in the weak nuclear interaction. The entangled states expected from these works were equivalent to the two-qubit version of the EPR state of the type discussed previously but using instead the states of horizontal and vertical polarization discussed in Chapter 2, $|\psi_{AB}\rangle_{EPR} = (|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)/\sqrt{2}$. However, none of these works were presented in the context of EPR or referenced the EPR paper. It was an article by Bohm and Aharonov [484] seven years later that discussed EPR in reference to using an annihilation process to produce polarization of two quanta propagating in opposite directions from the Wu-Shaknov paper. Bohm and Aharonov would say that the results of EPR could be tested by polarization properties of pairs of photons. They pointed out that the magnitude of the Wu-Shaknov result agrees with quantum mechanics and disagrees with the Schrödinger-Furry hypothesis. The Bohm-Aharonov paper led subsequent researchers to use optical polarization based measurements to test EPR correlations [83] [85]. All of these experiments verified the expectations of quantum mechanics for EPR entangled states. Thus it took around two decades before the EPR scenario began to be realized. Although EPR originally proposed their state via momentum conservation and the first experimental evidence was obtained via photon polarization, EPR states are now commonplace in quantum optics and quantum information. Entanglement distribution has been demonstrated over more than 1200 kilometers [485] with no evidence of the Schrödinger-Furry effect. EPR states today appear via experimental processes such as spontaneous parametric down conversion, spontaneous emission, Raman scattering, ionization, etc., all verifying the predictions of quantum mechanics [486], Figure 5.17.

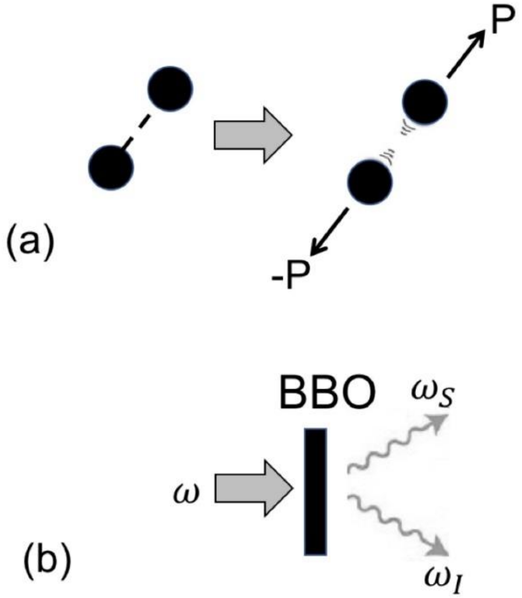

Figure 5.17 EPR state formation (a) Particle momentum conservation
(b) Photon pairs from spontaneous parametric down-conversion [486].