Johnny Goes to Göttingen

At the time of the EPR and Schrödinger papers in 1935-1936, the renowned mathematician and polymath John von Neumann was also at the Institute for Advanced Study (IAS) along with Einstein. In addition to his many fundamental contributions to mathematics, mathematical physics and computer science, von Neumann had also published *The Mathematical Foundations of Quantum Mechanics* [13] in 1932, a tour de force which had formulated quantum mechanics with mathematical rigor using the theory of Hermitian operators and Hilbert spaces as well as introducing the *Measurement Postulate* as a means of implementing the non-unitary aspect of the measurement process. The existence of a single unpublished letter dated April 11, 1936, from von Neumann to Schrödinger gives evidence that Einstein actually had contact with von Neumann regarding quantum mechanics and shared with him some of his correspondence from Schrödinger [487, pp. 211-213]. The letter to Schrödinger begins,

> *Einstein has kindly shown me your letter as well as a copy of the Pr. Cambr. Phil. Soc. manuscript.*
>
> John von Neumann: Selected Letters, 2005, p. 211, J. von Neumann, appearing by permission of the American Mathematical Society.

This refers to the second of the "Probability Relations between Separated Systems" papers [464] in which Schrödinger had introduced steering. Von Neumann would certainly have readily understood the details of Schrödinger's developments regarding the entangled states since he indicated that he was glad that Schrödinger had liked the mixing method. Schrödinger had evidently followed von Neumann's treatment of mixed states from his 1932 book. The mathematical theory of quantum mechanics in von Neumann's book also allows multi-particle entangled states to be analyzed and characterized in terms of their individual subsystems, though von Neumann did not categorize the properties of entanglement as Schrödinger had done in his 1935-1936 papers.

However, von Neumann does not appear to perceive the resulting quantum nonlocality demonstrated by Schrödinger as anything new or problematic. He objects to Schrödinger's concerns about the instantaneous correlations since such correlations can also exist classically,

> *I cannot accept your 4. completely. I think that the difficulties you hint at are "pseudo-problems." The "action at a distance" in the case under consideration says only that even if there is no dynamical interaction between two systems (e.g., because they are far removed from each other), the systems can display statistical correlations. This is not at all specific for quantum mechanics, it happens classically as well.*
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> John von Neumann: Selected Letters, 2005, p. 212, J. von Neumann, appearing by permission of the American Mathematical Society.

Von Neumann then described a simple classical example of two sealed boxes which both contain either one white ball each and or one black ball each but one does not know which color the balls are. The boxes are then spatially separated with one remaining on Earth and the second taken to Sirius. Of course, on opening the box on Earth, one immediately knows the state of the box on Sirius. To make the point, von Neumann uses the risqué joke: The moment something happened to his wife in Paris, the colonel was cuckolded in Madagascar. This type of correlation is now often called Reichenbach's *Principle of Common Cause* [488], that whenever two events are correlated, either one is a direct cause of the other, or they share a common cause. However, this principle is not straightforward when the correlated events A and B are space-like separated and in conjunction with relativistic causality. Of course, this was the situation later addressed by Bell [159] which considered a common cause in the past of A and B, denoted by hidden variables λ. This line of reasoning leads to the Bell inequalities which are violated by quantum theory and by experiments on systems with EPR correlations. Hence, von Neumann's explanation is incorrect. The principle of common cause plays a subtle but crucial role in the derivation of Bell's inequalities. Attempts at retaining some form of common cause in the face of quantum correlations result in a range of difficulties for any theory [191]. However, this was not known at the time of the letter, thirty years earlier.

Von Neumann's book had included his well-known theorem which demonstrated that hidden variables cannot correspond to operators in Hilbert space. In the proof, he assumes that a linear operator on a Hilbert space is associated with each measurement apparatus and that measurement results are given by eigenvalues of this operator. On this basis, he could show that, within the structure of quantum mechanics, there cannot exist any additional or hidden variables that could be used to distinguish statistical differences between particles in the same pure state. In this sense, quantum mechanics is a complete theory and von Neumann concluded that [13, p. 327],

> *The only formal theory existing at the present time which orders and summarizes our experiences in this area in a half-way satisfactory manner, i.e., quantum mechanics, is in compelling logical contradiction with causality. Of course, it would be an exaggeration to maintain that causality has thereby been done away with: quantum mechanics has, in its present form, several serious lacunae, and it may be even that it is false, although this latter possibility is highly unlikely, in the face of its startling capacity in the qualitative explanation of general problems and in the quantitative calculation of special ones.*

He was well-aware that his proof did not rule out hidden variables in general but that the link between physical quantities and operators would then have to be severed in that case. In his book [489, p. 89], Abner Shimony relates a story by Peter Bergmann, one of Einstein's assistants at the IAS, that Einstein was aware of von Neumann's analysis and once opened von Neumann's book to the page where the proof is given and pointed to the linearity assumption underlying the proof, asking "Why should we believe in that?" In 1952, David Bohm (1917-1992) had resurrected a thirty-year old approach of de Broglie's, though without being aware of it, and took advantage of the fact that von Neumann's theorem concerned hidden-variables theories of only a specific kind to develop contextual models with hidden variables not obeying that same linearity assumption but which could reproduce the predictions of quantum mechanics. It is sometimes stated that von Neumann's linearity assumption was unnecessary, therefore, his impossibility proof is mistaken; e.g., Bell's statement that, "There is nothing to it. It's *not* just flawed, it's *silly*!" [490]. However, this misconstrues von Neumann's carefully worded statement of the intent of his proof, i.e., that hidden variables cannot correspond to operators in Hilbert space [491]. In his book, von Neumann had concluded that the Hilbert space formalism is well supported but a formal proof cannot exist that no more complete theory than quantum mechanics will ever be possible. Thus, it's not surprising that Bohm had said in a letter to Pauli [492],

See the print edition of The Quantum Measurement Problem for quotation.

It's interesting then to consider in hindsight what von Neumann may have been thinking in his 1936 letter to Schrödinger concerning the white and black balls on Sirius.

Following Bohr's presentation at the *New Theories in Physics* conference in Warsaw in 1938, von Neumann gave an unscheduled presentation following Bohr, which included a succinct summary of his hidden variables impossibility proof as well as a discussion of quantum logic [493]. Afterwards, Bohr pointed out that the very simple experimental cases which he had alluded to in his own presentation showed, in more elementary form, the same essential points as those which appeared in von Neumann's mathematical analysis [493, p. 38]. Thus, Bohr was publicly comfortable with von Neumann's approach and that it was consistent with his own complementarity views. However, Bohr privately had impatience with formalism, and he had been seen shaking his head skeptically regarding von Neumann's proof [494, p. 263].

The first mention in print of the collapse of the state vector in quantum mechanics occurs in von Neumann's 1927 paper, *The Probability-theoretical Construction of Quantum Mechanics* [495] [496],

See the print edition of The Quantum Measurement Problem for quotation.

We have seen how the interrelated properties of nondeterminism, entanglement, and exact conservation laws are crucial to characterizing the measurement problem. The conservation laws had been experimentally shown to hold exactly for individual processes, beginning with the important Compton-Simon scattering experiment. Born was led to his probabilistic rule to account for measurement by the analysis of scattering processes. Von Neumann also used the Compton-Simon scattering experiments as the basis for a thought-experiment leading to the proposed *Measurement Postulate* in his 1932 book *The Mathematical Foundations of Quantum Mechanics* [13, pp. 212-217]. The Compton-Simon paper is the only experimental reference in the book. The exact conservation laws found by Compton-Simon are used as basis of von Neumann's argument [497], Figure 5.18.

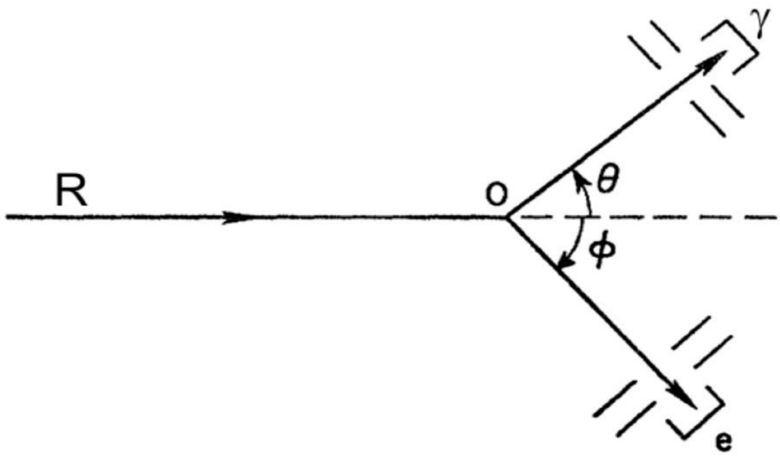

Figure 5.18: von Neumann's motivation for the projection postulate based on Compton-Simon cloud chamber experiments: (1) Central line R of the γ-e recoil collision is unknown but with a distribution of possible directions, conserved due to momentum conservation and (2) recoil of a photon γ and electron e toward detectors. If measurement of the photon γ occurs first, this is sufficient to determine R and therefore e, transitioning from statistical possibilities to a definite state of the electron e [497].
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The Compton-Simon experiment involved a monochromatic beam of X-rays passing through a cloud chamber, a vapor-filled vessel. Photographs of the interactions were taken at intervals and a pattern of tracks revealed the result of photons scattering from the orbital electrons of the molecules of the vapor. It was from such photographs that the validity of the conservation laws was deduced. It should be emphasized that the conservation laws also hold for relativistic particles in the relevant reference frame as is the case for these processes. Von Neumann considered a representative collision with the central line R of the collision along with the recoiling photon and electron each of which is measured by a detector, Figure 5.18. The central line is not known before the measurement, but the exact conservation laws enable us to determine R once the path of either recoiling particle is known. Therefore, before the measurement occurs, we can only make statistical statements about R. Suppose the photon reaches its detector first in our reference frame. Then due to exact conservation, the central line R is immediately known and therefore the state of the electron. The electron can be then considered as transitioning from a set of statistical possibilities to a definite state, a discontinuous and instantaneous reduction of the state. The electron momentum after the collision is given by $\acute{p}_e = p_\gamma - \acute{p}_\gamma + p_e$, yielding a transition of the electron to the momentum eigenstate $|\acute{p}_e\rangle$.

As part of his formulation, von Neumann proposed two postulates for evolution of a quantum system.

- Process 1: acausal non-unitary reduction applied for measurement
- Process 2: casual Schrödinger unitary evolution otherwise

Von Neumann's mathematical theory also allowed multi-particle entangled states to be analyzed and characterized in terms of their individual subsystems. As discussed in Chapters 3 and 4, non-unitary and unitary processes can be distinguished as a criterion for defining the measurement problem and entanglement plays a central role. Von Neumann makes the comment [13, pp. 418-420]:

> *Quantum mechanics describes the events which occur in the observed portions of the world, so long as they do not interact with the observing portion, with the aid of [process 2] evolution, but as soon as such an interaction occurs, i.e., a measurement, it requires the application of [process 1] evolution. The dual form is therefore justified.*
>
> *The difference between these two processes $\rho \rightarrow \acute{\rho}$ is a very fundamental one: aside from the different behaviors in regard to the principle of causality they are also different in that the former [Process 2] is (thermodynamically) reversible, while the latter [Process 1] is not.*

A von Neumann measurement is carried out in terms of the spectral decomposition of an observable A so that $A = \sum_i a_i P_i$ with projection operator $P_i = |a_i\rangle\langle a_i|$ for eigenstates $|a_i >$ and the non-degenerate eigenvalue a_i is found as a measurement result. Then the state vector makes a transition from the initial state to the corresponding eigenvector:

$$|\psi >= \sum_k c_k |a_k\rangle \longrightarrow |a_i\rangle.$$

In case of degeneracy, this was later generalized by Lüders in 1951 [498] to the projection

$$|\psi >= \sum_k c_k |a_k\rangle \longrightarrow \frac{P_i|\psi\rangle}{||P_i|\psi\rangle||}$$

with $P_i = \sum_k |a_{ik}\rangle\langle a_{ik}|$ being the projection operator on the subspace spanned by the eigenvectors of A for which $a_{ik} = a_i$. The von Neumann formalism has been generalized by Kraus and to quantum trajectories as discussed in Appendix 5.A.

The road toward developing von Neumann's theory began in the fall of 1926, when the prodigious twenty-two-year-old John von Neumann ("Johnny", younger than Dirac, Heisenberg, and Pauli) arrived at Göttingen University in Germany, a renowned center for mathematics. Its history had included Gauss, Riemann, Dirichlet, and Minkowski and now the driving force was the great mathematician David Hilbert. Max Born was an assistant to Hilbert when von Neumann joined the group. An eminent mathematician of his time, Hilbert also took a great interest in problems of physics and wanted to bring the rigor of mathematics to them and in particular use the axiomatic method, saying "physics is too hard for physicists." As a result, Hilbert gave courses and seminars in physics each year. Heisenberg had previously been Born's assistant and in 1925, the year he had discovered quantum mechanics at Niels Bohr's Copenhagen Institute, he presented his theory at Hilbert's seminar. This led to a burst of activity by Hilbert's group, furthering their interest in quantum mechanics, and initially intending to improve the existing formulations.

However, von Neumann realized that the natural framework for quantum mechanics was given by the abstract theory of separable Hilbert spaces and the associated linear operators. In this formulation, states of the physical system are given by Hilbert space vectors and measurable observables by Hermitian operators. This led to a two-year effort (1927-1929) by von Neumann, analyzing the formal aspects and the interpretation of quantum mechanics and this was developed in an axiomatic way [496]. It also led von Neumann into the mathematics of operator theory in which he achieved prominent results. Important physical quantities such as position and momentum are represented by unbounded Hermitian operators and by 1929 von Neumann had completely solved the problem of extending spectral theory to the unbounded case. In this way, von Neumann completely avoided the methods used by Dirac (including the singular Dirac delta-functions) which he had said "does not meet the demands of mathematical rigor in any way." Among other results, his approach showed the equivalence between matrix mechanics and wave mechanics in a rigorous way. Heisenberg's observables were operators on $\ell^2(N)$ (the space of complex square summable sequences) whereas Schrödinger's wave-functions were unit vectors in $L^2(\mathbb{R}^3)$ (the space of complex square integrable functions), and the inner products provide Born probabilities. A unitary transformation between these Hilbert spaces exhibits the mathematical equivalence between the two versions of quantum mechanics. Von Neumann also introduced the formalism of density operators $\hat{\rho}$ to characterize ensembles and *mixtures*, finding that the expectation value of an operator $\hat{A}$ is given by $\mathrm{Tr}(\hat{\rho}\,\hat{A})$ which holds for both pure and mixed states. The assumptions that went into recovering the Born rule were essentially the same as those for the no-hidden-variables proof [496].

In 1927, von Neumann published a trilogy of papers in the Göttingen Journal of Mathematics in which he developed the rigorous mathematical formulation of quantum mechanics. These became the basis of his 1932 book, *The Mathematical Foundations of Quantum Theory*, which was translated from German into English in 1949. The first three chapters essentially constitute the material from the trilogy, Chapter 4 deals with the completeness of quantum mechanics (i.e., the no-hidden-variables theorem), while Chapters 5 and 6 are devoted to measurement including irreversibility, thermodynamics, and the details of the measurement process. This included defining the entropy of quantum ensembles, addressing the irreversibility of macroscopic measurements due to the reduction of the wave function and the consistency of measurement regarding the interactions between a system and a measuring apparatus.

Von Neumann focuses in Chapter 6 on determining the precise point at which systems switch evolutions between Process 1 and Process 2, saying [13, p. 418]

> *…subjective perception is a new entity relative to the physical environment and is not reducible to the latter. Indeed, subjective perception leads us into the intellectual inner life of the individual, which is extra-observational by its very nature.*

In an example of the measurement of temperature, he gives four options for designating an *observer*: the thermometer, the human taking part in the measurement, the retina of the human, and the brain cells of the human. He concludes that the choice of designating the *observer* is arbitrary and a matter of convenience [13, p. 421],

> *In order to discuss this, let us divide the world into three parts: I, II, III. Let I be the system actually observed, II the measuring instrument, and III the actual observer. It is to be shown that the boundary can just as well be drawn between I and II + III as between I + II and III. (In our example above, in comparison of the first and second cases, I was the system to be observed, II the thermometer, and III the light plus the observer; in the comparison of the second and third cases, I was the system to be observed plus the thermometer, II the light plus the eye of the observer, III the observer, from the retina on; in the comparison of the third and fourth cases, I was everything up to the retina of the observer, II his retina, nerve tracts and brain, III his abstract "ego".)*

Within the framework of his axiomatic exposition of quantum mechanics, von Neumann then encounters the same issue as had the various *Copenhagen* narratives of quantum mechanics from Bohr, Heisenberg and Pauli: that of the dividing line between system and measurement device along with their quantum vs classical characteristics. As previously noted, Bohr, Heisenberg and Pauli shared a somewhat common set of views, sometimes known as the Copenhagen (or "orthodox") interpretation of quantum mechanics, though the persistence of a "Copenhagen viewpoint" may have owed much to Heisenberg's proselytizing for it during the 1950s [499]. More accurately, it could be described as a Copenhagen "spirit" rather than "interpretation" [500].

As emphasized by Zinkernagel [501], an issue from the beginnings of quantum mechanics is understanding the process of acquiring knowledge involving a separation between the observer and the observed system, this *cut* that allows a "detached" position for the observer to read at leisure the records left by the observed system. In a letter to Bohr, Pauli had called [213]

See the print edition of The Quantum Measurement Problem for quotation.

Bohr discusses how complementarity unexpectedly embraces the detached observer:

See the print edition of The Quantum Measurement Problem for quotation.

The cut can be made in many ways. Bohr's complementarity meant that the dividing line is determined by the nature of the experimental arrangement. However, Heisenberg and Pauli argued, and demonstrated by calculations, that the cut could be moved freely in the direction of the observer but that it cannot be moved arbitrarily far in the direction of the measured system. Heisenberg had used the existence and flexibility of the cut as the basis for an argument that hidden-variables cannot exist in quantum mechanics [502]. Since the laws of quantum mechanics apply to all systems, including the measuring apparatus, any steps to give a more complete account of the quantum state of a system would prevent the arbitrariness of the cut. Heisenberg appears to relate the cut to the necessity of eliminating what would come to be known as entanglement (i.e., "fine connections") between the system and the measurement device [501]:

See the print edition of The Quantum Measurement Problem for quotation.

As discussed in Chapter 4, Heisenberg also identifies two subprocesses: 1) where the system state becomes mixed because it interacts with a device and 2) when the state collapses. Subprocess 2) only occurs once an observer becomes conscious of it.