Non-Disturbing Measurement

The concept of back-action evading measurement can be extended to include two measurements. Given two observables *A* and *B* and an implementation that has average mapping $T_A(\varrho)$, *A* does not disturb *B* if for all $\varrho \in \mathfrak{M}(\mathcal{H})$ and all j $\in \Omega_B$ the probability of measuring B_j is the same whether or not *A* is measured previously to B_j. The condition for non-disturbing measurement is $\mathrm{Tr}\left[B_j T_A(\varrho)\right] = \mathrm{Tr}\left[B_j \varrho\right]$.

The condition for non-disturbing measurement can be rewritten in terms of the dual map of *A*, $T_A^*\left(B_j\right) = B_j$ and $j \in \Omega_B$ [229]. Note by setting *B*=*A*, one obtains a BAE measurement.

It has been proven in [659] that for finite dimensional Hilbert spaces *A* does not disturb *B* only if A and B commute. It has been noted [229] that in an infinite dimensional space there exist non-commuting observables for which *A* does not disturb *B*.

The following results are established in [229]:

1. If *A* does not disturb *B*, then *A* and *B* are jointly measurable. However, there exist examples for which *A* and *B* are jointly measurable but *A* does disturb *B*.

2. Given a POVM observable *B* for which $B_i^2 \in \overline{\mathrm{span}}(B), \forall i \in \Omega_B$, there exist implementations of *A* that do not disturb *B* if and only if *A* and *B* commute, where span(*B*) denotes the set of linear combinations of the vectors in set *B*.

3. Given the generalized Lüder's implementation for A, $\mathrm{T}_{\mathrm{A}}^{\mathrm{L}}(\varrho)$, if *A* and *B* commute then *A* does not disturb *B* (See Exercise 7.5 in the book or kindle version of theQMP)

4. Let *B* be a POVM observable such that $\mathcal{D}\left(\mathrm{span}(B)\right) \geq (d-1)^2 + 1$, where $\mathcal{D}$(*A*) denotes the dimension of the linear vector space *A*, and $\mathcal{D}(\mathcal{H}) = d$. There exist implementations of *A* that do not disturb *B* if and only if *A* and *B* commute. In the case where $\mathcal{D}(\mathcal{H}) = 2$, there exist implementations of *A* that do not disturb *B*, and there exist implementations of *B* that do not disturb *A*, if and only if *A* and *B* commute. In the case when $\mathcal{R}(B)=1$, where $\mathcal{R}(B)$ denotes the rank of *B*, there exist implementations of *A* that do not disturb *B* if and only if *A* and *B* commute.