Non-Demolition Measurement

Consider a system coupled to an apparatus initially in the state $\rho_S \otimes \rho_A$. A measurement operator X operates on the system during a time period $[t_{0,} t_1]$ for which the operator correctly correlates with the input state and is a repeatable measurement. The measurement operator X is a quantum non-demolition (QND) observable if the system is projected into an eigenspace corresponding to a distinct eigenvalue of the observable and not absorbed or removed in the process of measurement [665].

A sufficient condition for a QND [186, p. 39] is that the operator X in the Heisenberg representation is a constant of motion i.e. assuming a time-invariant Hamiltonian that couples the system and apparatus,

$$X \otimes I = U(t)^{\dagger}(X \otimes I)U(t).$$

However, it is not necessary that the observable be a constant of motion in order to qualify as a QND measurement as shown in [666]. Rather the condition given in [666] is that the observable X commute with itself at times when the observable is measured, i.e., $[X(t), X(t')] = 0$, where $t, t' \in [t_0, t_1]$.

The theory of continuous non-demolition measurement has been under development by several groups. The Belavkin equation is a stochastic approach that is often referred to as a quantum filter and has been applied to continuous non-demolition measurement [667]. Additionally, Korotkov and others have been active developing and applying theory to describe continuous measurement, see for example [668].