## *Repeatable Measurement*

Suppose a measurement of a system is made initially in state $\rho$ via a POVM observable A that is implemented via $\mathcal{E}_k(\varrho)$. If the outcome of the measurement is $i \in \Omega_A$, a repeatable measurement gives the same outcome $i \in \Omega_A$. After the first measurement, the probability of the $i$th outcome $P(i)$ is unity if the measurement is repeatable, that is $P(i) = \mathrm{Tr}[\mathcal{E}_i(\varrho)\, E_i] = 1$, provided $\mathrm{Tr}[\varrho\, E_i] > 0$.

A measurement is repeatable only if it is not continuous [661]. An equivalent condition for repeatability is that the dual map $\mathcal{E}_l^*(A_m) = 0$ for all $l \neq m$ [229]. There exists an implementation $\mathcal{E}_k(\varrho)$ of a POVM observable $A$ that is a repeatable measurement if and only if each $A_i$ has an eigenvalue of unity [662].

If $A$ is a repeatable POVM observable and $\mathcal{D}(\mathcal{H}) = d < \infty$ has an outcome space with cardinality given by $|\Omega_A| = d$, then $A$ is sharp, where the dimension of the Hilbert space $\mathcal{H}$ is denoted by $\mathcal{D}(\mathcal{H})$. If $|\Omega_A| \geq d - 1$ then $A$ is commutative [229].